Problem Creator: Máté Vigh

Eka Tjipta Foundation Problem
Theoretical 1: Motion of a Rolling Rod

In this problem, the motion of a uniform rod (stick) with length L, ended with caster-wheels at both ends, will be investigated on a flat surface. The casters at each end of the rod can spin freely and independently (see Figure 1) and have a negligible mass compared to the rod. The friction between the rod and the caster-wheels is negligible. The diameter of the caster-wheels are a bit larger than the diameter of the rod, but both diameters are much smaller than the length of the rod. The gravitational acceleration is g.

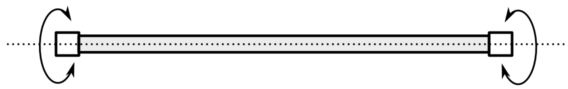

Figure 1: Sketch of the rod with the caster-wheels.

1. The rod is placed on a horizontal flat surface and pushed such that each end of the rod get different horizontal initial velocity (v_1 and v_2, pointing in the same direction) perpendicular to the axis of the rod. The casters roll without slipping on the surface.
   (a) Calculate the initial velocity v_0 of the center of the rod and the initial angular velocity ω_0 of the rod using v_1, v_2 and L! (0.8 points)
   (b) Describe the motion of the center of mass of the rod! Determine the parameter(s) of its orbit! (0.8 points)
   (c) What should be the minimum value of the coefficient of static friction μ for the casters to not slip on the surface? (0.6 points)

   In the following sections the case of the inclined surface will be considered. The angle between the inclined surface and the horizontal plane is α.

2. If α is infinitesimally small, the motion of the rod slightly changes: the motion of the center of mass is approximately the same as in the previous section but a constant drift velocity v_{drift} added to the solution. Use a coordinate system as in Figure 2.
   (a) Calculate the magnitude and the direction of v_{drift} as a function of the small α, the initial velocities of the two ends of the rod (v_1 and v_2, pointing in the same direction) and the gravitational acceleration g! (1.9 points)
   (b) Sketch the orbit of the center of mass of the rod! (0.5 points)

3. If α is finite, the details of the motion of the rod changes. Place the rod on the inclined plane along the steepest line of the surface (so the rod is parallel with the inclined edges of the plane). Consider that the initial velocity v_0 of the center of mass of the rod is perpendicular to the axis of the rod and the initial angular velocity ω_0 is perpendicular to the surface as shown in Figure 2.
   (a) Calculate the time evolution of the velocity $\mathbf{v}(t) = (v_x(t), v_y(t))$ of the center of mass of the rod in the Cartesian coordinate system shown in Figure 2. (0.8 points)

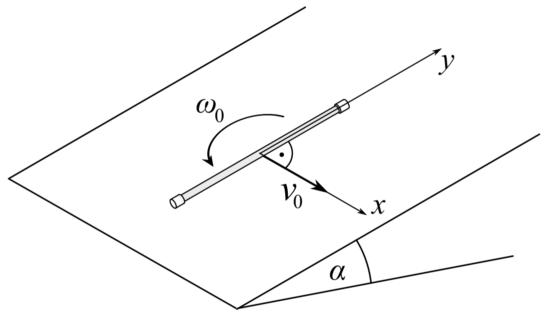

Figure 2: The initial conditions of the rod in section **3.**

(b) Depending on the magnitude of the v_0 and ω_0, it can occur, that the center of the rod stops for a moment during its motion. Express the condition(s) for such a behavior using the parameters v_0, ω_0, g, α and L! (0.8 points)

(c) Determine the maximum displacement of the center of the rod in the direction of the steepest line (y-direction) as function of v_0 and ω_0! (1.2 points)

4. Investigate another situation where the rod is placed horizontally on the inclined surface. Consider that the initial angular velocity ω_0 of the rod is perpendicular to the surface but the initial velocity of the center of the rod is zero (see Figure 3).

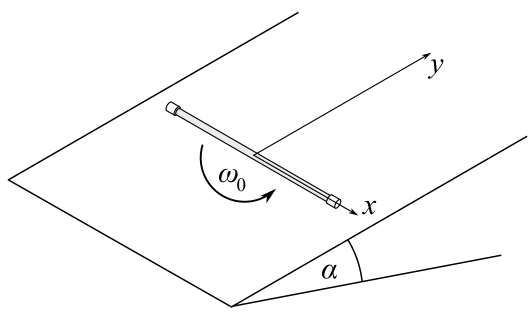

Figure 3: The initial conditions of the rod in section **4.**

(a) Describe the motion of the center of mass of the rod! Determine the parameter(s) of its orbit! (1.6 points)

(b) What should be the minimum value of the coefficient of static friction μ in this case for the casters to not slip on the surface? (1.0 points)