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IPEKA Problem
Theoretical 2: Why Maglev Trains Levitate

Maglev is a technology for magnetic suspension (levitation) and propulsion of trains or other vehicles. Since there is no friction force between the train and the rails, Maglev trains reach record velocities approaching 600 km/h.

There are three types of Maglev systems – EMS (Electromagnetic Suspension), EDS (Electrodynamic Suspension), and the experimental Inductrack technology. In this problem you are going to explore the physical principles of Inductrack suspension on a simplified model of a Maglev train. The principle of magnetic propulsion will not be considered here because it has a lot in common with the physics of magnetic levitation.

Shown in Figure 1 is a schematic side view of an Inductrack train-car. When at rest or moving at a low speed, the car lies with its wheels on the rails like any ordinary train. The car, however, detaches from the rails at a specific takeoff velocity v_t due to the system described below: two long parallel arrays of permanent cubic-shape magnets of size $a = 5\ cm$, each is located at the bottom of the car. The magnetic dipole moments of the neighboring magnets are tilted at an angle of $45°$ relative to each other (the so called *Halbach* array). As a result a static magnetic field is produced below each array with components of the magnetic field given by the equations:

$$B_x = B_0 \exp(-ky) \sin(kx) \tag{1}$$

$$B_y = B_0 \exp(-ky) \cos(kx) \tag{2}$$

where the x-coordinate is measured from the rear end of the car in the direction of motion and y-coordinate from the bottom of the car in a downward direction. The parameter k is the wavenumber of the magnetic field. The amplitude of the magnetic field is $B_0 = 1.4\ T$.
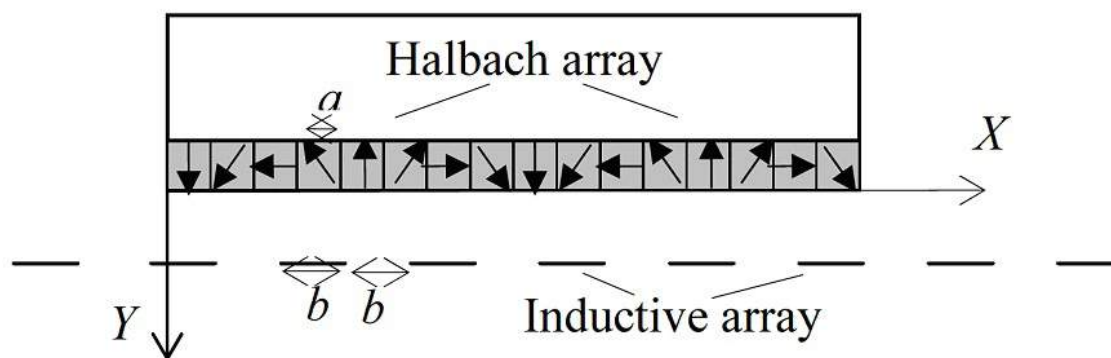

Figure 1: A schematic side view of an Inductrack train-car. The arrows show the directions of magnetic moments of the cubic magnets in the *Halbach* array. The wheels and the rails of the train are not shown for convenience.

Two *inductive arrays* of horizontal rectangular wire frames of the same width a as that of the permanent magnets are arranged along the rail as seen from the top view in Figure 2. The length of the inductive frames as well as the distance between them is b. Each inductive array is located below the corresponding Halbach array at a distance corresponding to the y-coordinate of the inductive array (see Figure 1). Assume that the equations 1 and 2 for the components of

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the magnetic field hold only for the inductive frames, which are located below the car, while the magnetic field outside the car area is strictly zero.
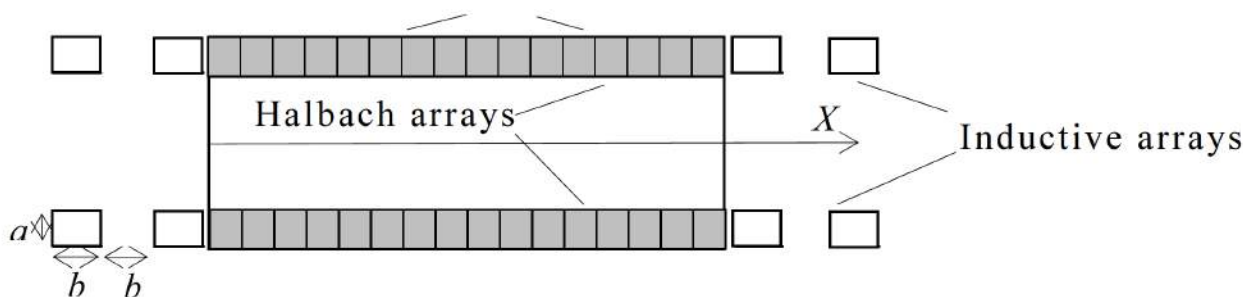

Figure 2: A schematic top-view of an Inductrack train-car. The wheels and the rails of the train are not shown for convenience.

Note: In the real Inductrack trains, the frames of the inductive array are electrically connected and form a continuous ladder-like array. In order to simplify the theoretical consideration, we use a simplified model shown in Figure 2.

Helpful mathematics formulas:

$$\sin(x) - \sin(y) = 2\sin(\frac{x-y}{2})\cos(\frac{x+y}{2}) \tag{3}$$

$$\cos(x) - \cos(y) = 2\sin(\frac{y-x}{2})\sin(\frac{x+y}{2}) \tag{4}$$

In parts 1-2 assume that the train car is infinitely long (*i.e.* equations 1 and 2 hold for all values of the x-coordinate)

1   Electromotive force in the inductive wires

(a) Derive a relationship between the wavenumber of the magnetic field k and the lateral size of the cubic magnets in the Halbach array a. Calculate k numerically. (0.3 points)

(b) Suppose that the train moves with a constant velocity v in the positive direction of the x-axis. Consider an inductive frame whose center in the initial moment ($t = 0$) is located below the car at a point of coordinate x_c relative to the car. Derive an expression for the electromotive force (EMF) ε induced in the frame as a function of the time t and the other parameters already defined. (1.5 points)

(c) What is the angular frequency ω of the induced EMF. (0.4 points)

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2 Current in the inductive wires

Each frame in the inductive array is characterized by a self-inductance L and a resistance R. The mutual inductance between different frames is negligible compared to L. The current I induced in the frame varies with time according to the equation:

$$I(t) = I_0 \sin(\omega t - k x_c + \psi) \tag{5}$$

(a) Obtain expressions for the amplitude I_0 and the phase-shift ψ of the alternating current induced in the frame. (2.0 points)

Note: The positive direction of circulation of the induced current in the frame is related to the positive direction of the y-axis according to the right-hand rule.

3 The dynamics of the train

(a) Derive formulas for the time-averaged components $\bar{F}_x$ and $\bar{F}_y$ acting on a single inductive frame in terms of velocity v, the distance between the Halbach array and the inductive array y, and the others parameters already defined. Sketch qualitative graphs of F_x and F_y versus train velocity v for a fixed value of the distance y. (3.0 points)

(b) Consider a train-car of a large but finite length l ($l \gg b$). Derive expressions for the magnitude of the total lift (vertical) and drag (horizontal) forces F_L and F_D acting on the car.

(1.2 points)

(c) For a given value of a, what is the minimum aspect ratio b/a for which the lift force is maximum for given values of the distance y, velocity v, resistance R, and the inductance L. (1.0 points)

Consider a train-car of length $l = 10\ m$ and mass $m = 10000\ kg$. Assume that the aspect ratio b/a corresponds to the optimal value found in 3(c). The inductance and the resistance of the inductive frames are $L = 1.0 \times 10^{-7}\ H$ and $R = 1.0 \times 10^{-5}\ \Omega$ respectively. The acceleration due to gravity is $g = 9.8\ m/s^2$. Assume that the distance between the Halbach array and the inductive array is $y = 0\ m$, when the train is at rest.

(d) Obtain an expression and calculate the takeoff velocity v_t, the velocity at which the train detaches from the rails. (0.3 points)

(e) What is the distance y between the train and the rails at an operating velocity of $360\ km/h$?

(0.3 points)