Magnetic Dipole Oscillation

A magnetic dipole with magnetic moment m_1 is placed at the coordinate origin parallel to the x-axis.

1. Determine the resulting magnetic field in all space.
2. Another dipole is placed at a distance r from the origin at an angle θ to the x-axis. The magnetic moment of the second dipole, m_2, forms an angle α to the x-axis. The whole set-up can be seen in Fig. 1. Determine the torque on the second dipole.
3. Determine the interaction energy between the two dipoles.
4. Determine the force on the second dipole.
5. The second dipole is tied to the first dipole via a massless string such that the distance between the two is fixed at R. While the orientation of the first dipole at the coordinate origin is fixed, the orientation of the second dipole may change. It is also allowed to move freely in the xy-plane around the first dipole. Write down the equation of motion of the second dipole. The mass and moment of inertia of the second dipole are taken to be m and I respectively.
6. Initially the second dipole is at rest on the x-axis, with the magnetic moment forming an angle α_0 to the x-axis ($\alpha_0 << 1$). At $t = 0$, the second dipole is released and allowed to move freely. Write down the equation of motion of the second dipole assuming θ and α are small.
7. The system undergoes simple harmonic oscillation. You are asked to determine the normal mode frequencies of oscillation. The system is in a normal mode when the oscillating variables are in phase and can be written as follows: $\theta = \theta_0 \cos(\omega t + \phi)$ and $\alpha = \alpha_0 \cos(\omega t + \phi)$. There are two possible values of ω (denoted by ω_1 and ω_2). Determine ω_1 and ω_2.
8. For each normal mode, determine the ratio of the amplitude of α to θ, $c_1 = \alpha_1/\theta_1$ and $c_2 = \alpha_2/\theta_2$.
9. The equation of motion of the system can be expressed as follows:

$$\begin{aligned} \theta &= \theta_1 \cos(\omega_1 t + \phi_1) + \theta_2 \cos(\omega_2 t + \phi_2) \ , \\ \alpha &= c_1\theta_1 \cos(\omega_1 t + \phi_1) + c_2\theta_2 \cos(\omega_2 t + \phi_2) \ . \end{aligned}$$

   Using the initial conditions, determine the values of θ_1, ϕ_1, θ_2, and ϕ_2.

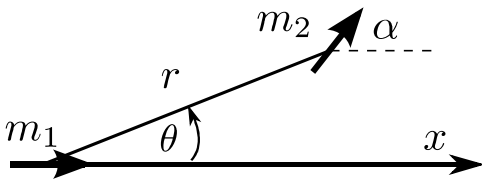

Figure 1: