Cylinder Collision

A hollow cylinder with mass M and radius R is at rest on a horizontal plane. In the interior of this cylinder, there is a solid disk with mass m and radius r. Initially, the center of the disk is at a distance l from the center of the cylinder and moves with velocity $v\,\hat{y}$ as shown in Fig. 1. Unless otherwise specified, all collisions are elastic and frictions can be ignored.

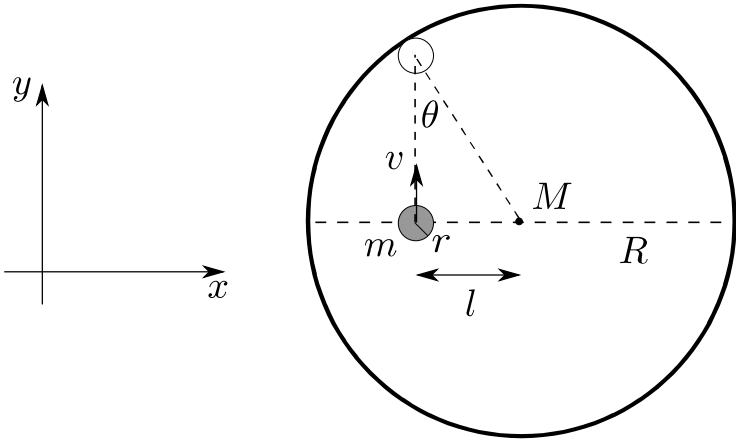

Figure 1:

1. Determine the velocity (the $\hat{x}$ and the $\hat{y}$ components of the velocity) of the disk and the cylinder immediately after the first collision. Write your answer in terms of m, M, v and θ.

2. Determine the velocity (the $\hat{x}$ and the $\hat{y}$ components of the velocity) of the disk and the cylinder immediately after the second collision. Write your answer in terms of m, M, v and θ.

3. If initially the disk is placed at $l = (R - r)/2$, determine the velocity of the disk and the cylinder immediately after the n-th collision.

4. What is the condition for l such that immediately after the n-th collision m moves with velocity $v\,\hat{y}$ and M is at rest? Determine the distance between two successive positions of the center of M when it is at rest.

5. For this part, the friction between the the disk and the cylinder cannot be ignored. As in part (a), initially the cylinder is at rest, while the center of the disk is at a distance $l < (R - r)$ from the center of the cylinder and moves with velocity $v\,\hat{y}$ as shown in Fig. 1. If during the collision process the point of contact does not slide, determine the angular velocity of the disk and the cylinder immediately after the first collision.