Problem 1 Electrostatic Balloon
Deadline March, 31 2012

In this question we will investigate various aspects of a hypothetical aerostatic flying machine which we shall call *electrostatic balloon*. The idea is to take some electrically conducting metal foil and shape it into a sphere of radius R (consider the foil to be essentially weightless and possess sufficient structural strength). Then evacuate it completely and in order to balance the atmospheric pressure, charge the balloon by a suitable electrostatic charge Q. The boyant force will arise from Archimedes' principle just as it happens with a conventional hot-air balloon, and the machine will be able to carry a payload of mass M.

Problems

To obtain full marks, give a succinct reasoning for your answers.

1. For a balloon of radius R and total charge Q, find
   (a) the surface charge σ,
   (b) the electric field $\mathbf{E}(\mathbf{r})$ in all space,
   (c) the potential V with respect to infinity,
   (d) the energy W required to charge the sphere.

   Take the permittivity of the surrounding air to be unity.

2. Find what surface charge σ will exactly counter-balance the atmospheric pressure P. You may proceed as follows:
   (a) Consider a small patch on a general charged conducting surface. Show that the effective electric field which acts on the patch is

   $$\mathbf{E}_{\text{effective}} = \frac{1}{2}(\mathbf{E}_{+} + \mathbf{E}_{-})$$

   where $\mathbf{E}_{+}$ and $\mathbf{E}_{-}$ are the electric fields immediately above and below the conducting surface, respectively. (*Hint*: treat separately the electric field produced by the small patch and that produced by all other charges.)
   (b) Find what σ is required so that the expanding electrostatic force balances the contracting atmospheric pressure.

3. We will now take into account that air has a small but non-zero conductivity s (the inverse of resistivity $\rho = 1/s$), which causes the balloon to discharge over time.
   (a) If the balloon is initially charged with Q_0 and left unattended, find how its charge will evolve with time $Q = Q(t)$.

(b) Suppose we install a charger which compensates the discharge due to air and keeps the charge constant at Q_0. How much power Π will it consume? Express it as a function of the balloon radius R only.

4. The whole purpose of the balloon is to carry a payload of mass M (in this problem neglect the mass of the balloon itself.) Find how the following quantities depend on M:

   (a) total charge Q,

   (b) energy W [refer to Problem 1.(d)],

   (c) power Π [Problem 3.(b)].

5. Consider a conventional hot-air balloon. Assume it to be spherical. A gas burner heats up the air, so that its density decreases inside the balloon and Archimedes force provides boyancy. Use physical reasoning and practical considerations to estimate how the required burner's power depends on the payload of mass M. Assume the volume of mass M do not contribute to the buoyant force. Comment on which kind of technology is better suited for flying. Any sound answer will be given credit.

6. In practice one can expect the electrostatic balloon to be not perfectly hermetic. Consequently, the vacuum inside will be slowly filled with air from the atmosphere. We can model the situation by imagining that in the balloon there are tiny holes with a total surface area $S \ll R^2$. Assume the volume of mass M do not contribute to the buoyant force.

   (a) Let the atmospheric pressure be P_1 and the pressure inside the balloon $P_2 \ll P_1$. Find the velocity of the gas v_2 when it enters the balloon through some tiny hole on the surface. Temperature of the atmosphere is T_1, adiabatic index γ, molar mass μ. [*Hints*: Make use of the Bernoulli equation (conservation of energy for a fluid element). Assume that the gas flows fast enough so that it does not have time to exchange any heat with the surroundings.]

   (b) We install a pump which keeps the balloon pressure at $P_2 \ll P_1$. Find the power consumed by such a pump and express it as a function of payload M. Assume the pump's hole size is equal to S, and $P_2 = 0$.