Deadline July, 31 2012

The Moon's gravitational pull is commonly known to be among main reasons for cyclic sea level changes, or tides. In this problem, you are asked to determine a shape of the ocean water surface and a tide height. To estimate the influence of the Moon, it will suffice to consider the simplest model of an Earth-Moon system (part 1). A detailed and correct description is to be based on a more sophisticated model (part 2).

The only objects considered in the problem are Earth and Moon. The influence of the Sun and the rest of planets in the solar system will be neglected (except for question 2.7 where the Earth-Sun system is considered). Assume also that the Earth surface is completely covered by water and the ocean floor doesn't prevent the water movement. You should solve the problem in terms of gravitational potential.

Potential of the gravitational field is the ratio between the potential energy of a material point placed into the gravitational field and the mass of the point. It is analogous to the electric potential.

The gravitational potential of the point mass m at a distance r from it is:

$$\varphi = -G\frac{m}{r} + const \tag{1}$$

where $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/s^2}$ gravitational constant.

The arbitrary constant is governed by conditions of the potential normalization. The value of the constant does not matter, so while solving the problem, one can drop all the constants from expressions for the gravitational potential.

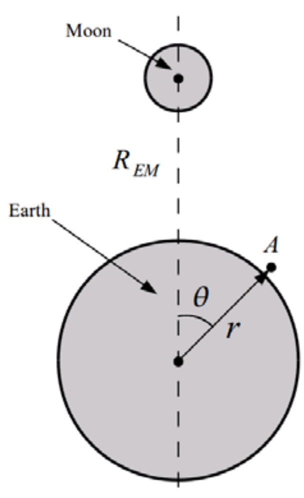

Figure 1: Earth and Moon

Earth–Moon system parameters:
Earth Mass : $M_E = 6.0 \times 10^{24}$ kg;
Moon Mass : $M_M = 7.4 \times 10^{22}$ kg;
Earth Radius : $R_E = 6.4 \times 10^6$ m;
Earth–Moon distance: $R_{EM} = 3.8 \times 10^8$ m;

Part 1. Simplest model

In the simplest model, the revolution of Earth and Moon about the barycentre and their rotation around axes are neglected. Assume that Moon and Earth are fixed and the distance between their centers is R_{EM} (fig. 1).

1.A. Write down the expressions for gravitational potentials $\varphi_E(r)$ and $\varphi_M(r, \theta)$ of the field created by Earth and Moon respectively in a point A located close to the Earth surface at a distance r from its centre and at an angle θ to the Earth–Moon line (fig. 1).

1.B. Using the approximation $(1+x)^\alpha \approx 1+\alpha x$, write down the expression for the resultant potential: $\varphi(r, \theta) = \varphi_E(r) + \varphi_M(r, \theta)$. Discard all constants.

1.C. The ocean water surface is equipotential. Which points at the Earth surface correspond to minimum and maximum water levels? Sketch a water surface shape, indicate the Moon location on your sketch.

1.D. The tidal height is a half of the difference between maximal and minimal marks of the water level. Calculate the value of the tidal height.

Part 2.More sophisticated and correct model.

In this part you should take account of the Earth and Moon orbiting around their barycentre. Neglect as before the rotation of the Earth about its axis.

2.A. Assuming that Earth and Moon move in circular orbits, find the distance d between the centre of Earth and the Earth–Moon system barycentre. Find the angular velocity ω of the system rotation. Express d and ω in terms of the Earth–Moon system parameters.

2.B. There is centrifugal force in the rotating frame of references. This force can be taken into account by introducing a centrifugal potential φ_C . Express the centrifugal potential in terms of both the distance x to the axis of rotation and the angular velocity ω.

2.C. Write down the expression for the centrifugal potential $\varphi_C(r, \theta)$ in the point A (fig. 1). As before, discard all the constants.

2.D. Write down the expression for the total potential, $\varphi(r, \theta) = \varphi_E(r) + \varphi_M(r, \theta) + \varphi_C(r, \theta)$. When writing $\varphi_M(r, \theta)$, use the approximation $(1+x)^\alpha \approx 1+\alpha x+\frac{\alpha(\alpha-1)}{2}x^2$ to retain the dependence on angle θ.

2.E. Which points at the Earth surface correspond to minimum and maximum water levels? Sketch a water surface shape; indicate the Moon location on your sketch.

2.F. Calculate the value of the tidal height.

2.G. Consider the Earth-Sun two-body system. Calculate the value of the tidal height caused by the Sun. The Sun mass: $M_s = 2.0 \times 10^{30}$ kg. The distance between the Earth and the Sun: $R_{ES} = 1.5 \times 10^{11}$ m.