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Theoretical 2: Physics of the Oil and Gas Production

It is a common misunderstanding that oil is located in the form of an underground lake. Actually, it is found in very small pores with a size comparable to the diameter of human hair. Void space between particles of sand is filled with oil like a sponge is filled with water. If one pushes on the sponge, water comes out. A similar thing occurs with oilsaturated rocks, which are located a few kilometers beneath the surface of the Earth. When all the rocks above the reservoir exert a huge pressure on the petroleum sponge, the oil flows to the surface (see Figure 1). In this problem we neglect capillary and gravity effects on the fluid flow.

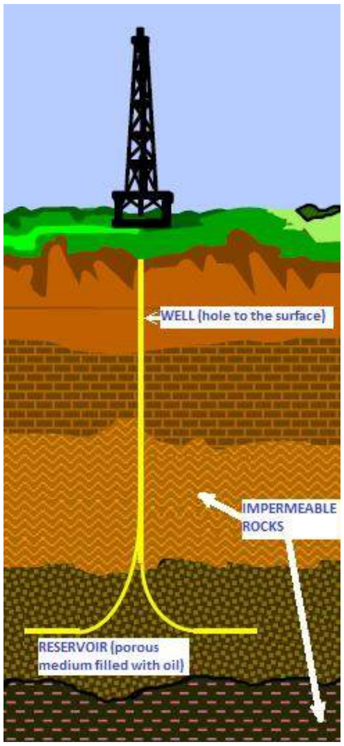

Figure 1: Scheme of the oil production process

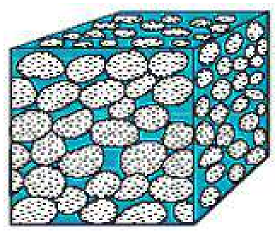

Figure 2: Representation of the porous medium (grains in white and void space in blue)

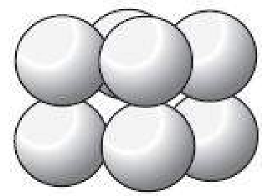

Figure 3: Cubic stacking of identical spherical grains

1 Basic Concepts

(a) One of the most important characteristics of the reservoir is porosity, which is the fraction of the void space in the rock to the total volume:

$$\varphi = \frac{V_{void}}{V_{grains} + V_{void}} \,, \tag{1}$$

where V stands for the volume.

To understand meaning of this concept, imagine identical balls (grains of sand) that put in a pile as shown in Figure 3.

Find the porosity of the system if the number of balls is infinitely large. (0.3 points)

(b) A fluid flow between the grains of sand is controlled by viscosity and permeability. Consider a flow of the viscous fluid through a tube with length L_0 and radius r_0 (Figure 4). Fluid molecules moves along free paths and collide with each other. However, this process is not uniform. Close to the solid boundary molecules are stuck, while in other regions velocity varies, with a profile similar to the sketch shown in Figure 4, where y is measured from the axis of the tube.

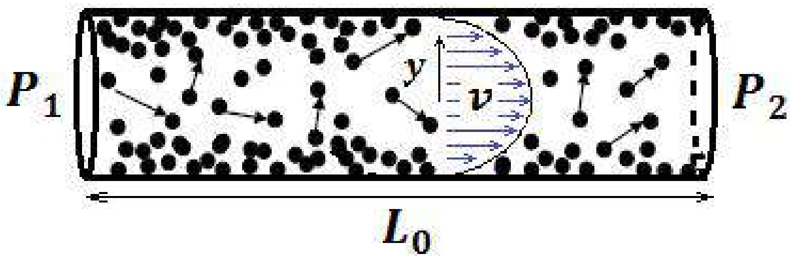

Figure 4: Schematic of the viscous fluid flow in the tube.

The reason for this effect is the internal friction of the fluid, or viscosity. If two adjacent layers of fluid flow with slightly different speeds, the random sidewise intrusion of some faster molecules into the slower stream will tend to speed up the slower stream, whereas intrusion of slower molecules into the faster stream will tend to slow down the faster stream. This effect could be quantified with following well-known equation:

$$F_{fr} = -\mu A_{fr} \frac{dv}{dy} \ , \tag{2}$$

where F_{fr} is a friction force which occurs between two thin layers of the fluid separated by a small distance dy, which have differences in velocities dv; A_{fr} is a contact area on which applied the internal friction force; μ is the fluid property called coefficient of viscosity.

Find the velocity distribution $v(y)$ in terms of μ, L_0, r_0, P_1 and P_2 . Assume that a mean free path of molecules is much smaller than the radius of the tube. (0.6 points)

(c) Under described conditions fluid will flow through the tube with a flow rate:

$$q = \frac{k_0}{\mu} \pi r_0^2 \frac{P_1 - P_2}{L_0}. \qquad (\textit{Poiseuille equation}) \tag{3}$$

Calculate coefficient k_0 in Poiseuille equation. (0.3 points)

(d) A fluid flow through a porous medium is governed by Darcy's law:

$$q = \frac{dV}{dt} = \frac{k}{\mu} A \frac{(P_{in} - P_{out})}{L} \ , \tag{4}$$

where $\frac{dV}{dt}$ is the amount of fluid transferred through the rock in some period of time; A, L are the cross-sectional area and length of the sample shown in Figure 5; $P_{in} - P_{out}$ is the pressure drop; k is the permeability and is the property of the rock

(*You can easily recognize some similarity with Fouriers Law for heat transfer. Using analogies with heat transfer could significantly help in solution of this problem, because approaches are very similar*).

Porous medium can be modeled as a system of twisted tubes (Figure 5), with permeability $k = k_0\varphi^2$ Where k_0 is permeability of a straight capillary; φ^2 accounts for nonlinearity of the tubes in a porous medium with porosity φ.

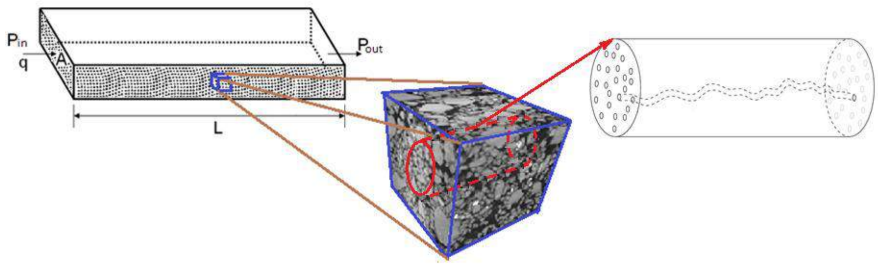

Figure 5: Diagaram showing definition of Darcy's law

Estimate permeability of the system described in 1.a, with radiuses of the balls equal to 10^{-6} m. (0.1 points)

(e) Usually, rock properties are not uniform throughout the reservoir. However, it is possible to apply an averaging procedure to find an effective permeability k_{eff}. This means that the initial system could be replaced with a new model that has the same sizes and fluid flow parameters with the only difference in permeability, which is uniform throughout the new homogeneous sample. To examine this issue, we consider a sample consisting of two different rock types as shown in Figure 6. An incompressible fluid flows through that system with a flow rate q and viscosity μ.

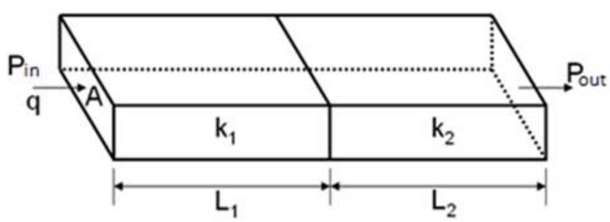

Figure 6: Composite rock sample

Calculate pressure at the boundary between two different rocks P_b in terms of q, μ, and parameters shown in Figure 6. (0.8 points)

(f) Find the effective permeability of the system k_{eff}. (0.4 points)

2 Vertical Well

Often the reservoir can be modeled as a cylinder (see Figure 7). For this problem all properties were averaged out as in the previous part, so the reservoir is assumed homogeneous with uniform permeability k. Oil can be viewed as an incompressible fluid with viscosity μ. Because the rocks above and below the reservoir are impermeable and the height of the cylinder is much less than its radius ($h << R$), one can conclude that the fluid flows only in the radial direction.

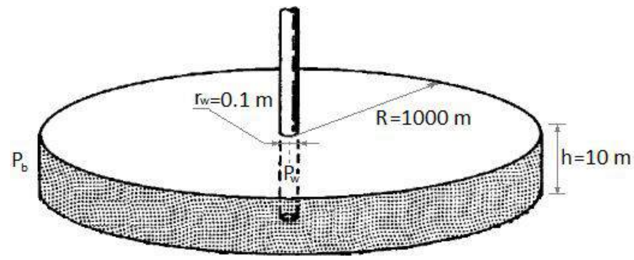

Figure 7: Cylindrical reservoir with a vertical well drilled in the center

(a) Find the velocity of the oil v_w inside the well with radius $r_w = 0.1$ m, if the flow rate is 30 $\frac{m^3}{day}$. Estimate the fluid velocity in the reservoir near the well v_r. (0.4 points)

(b) The calculated fluid velocity in the reservoir is rather small therefore the reservoir pressure can be treated as a constant for several months or even years, especially if the reservoir is connected with an underground source of water. Let P_b be the pressure at the outer boundary of the reservoir and P_w be the pressure at the bottom of the well. In this part assume that both P_b and P_w are constants (time-independent values), as well as the radial pressure distribution.

Calculate pressure drop $P_b - P_w$, which is required to produce oil with flow rate q.

(1.0 points)

(c) Make a sketch of pressure distribution in the reservoir $P(r)$ as a function of the distance from the well. (0.4 points)

3 Modeling Reservoir Depletion

In this part the depletion process will be analyzed for the reservoir shown in Figure 8. The well has a horizontal part, therefore, the fluid flow in the reservoir is linear ($h << L$).

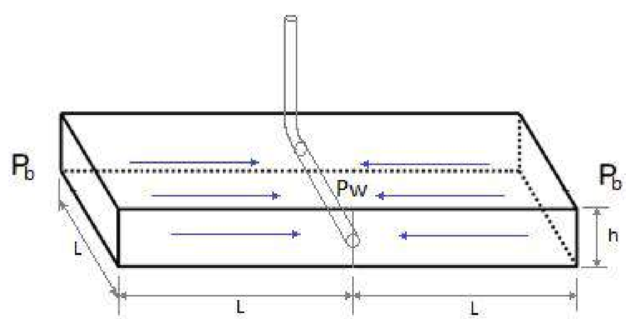

Figure 8: System used for modeling reservoir depletion

This time, pressure at the bottom of the well P_w is constant (hydrostatic column of oil). However, the pressure at the boundary $P_b(t)$ is changing with time, as well as the oil production rate

$q(t)$. In this model the flow rate of the fluid is given by:

$$q = -2\varphi L^2 c_r h \frac{d\bar{P}}{dt} \ , \tag{5}$$

where c_r is the compressibility of the rock, and the average pressure inside the reservoir $\bar{P}$ is approximated by:

$$\bar{P} \approx \frac{P_b + P_w}{2}. \tag{6}$$

(a) Derive an explicit expression for $q(t)$ in terms of k, μ, c_r, φ and reservoir dimensions, if the initial flow rate is q_0. (1.2 points)

(b) For the system described in this section, determine the time needed to deplete the oil reservoir by a half. (1.0 points)

(c) Historically, first attempts to predict reservoir performance were made by doing analogies to electric circuits. Draw a simple electric circuit, which is analogous to the system described in Part 3. (0.5 points)

(d) The reason why gas reservoirs cannot be modeled with the circuit as in 3.c, is that gas is highly compressible, with a compressibility being strongly dependent on the applied pressure. Compressibility is defined as:

$$c = -\frac{1}{V}\left(\frac{dV}{dP}\right)_T \ , \tag{7}$$

where V is initial volume of the examined sample, dV is isothermal volume change, when additional pressure dP is applied.

Assuming natural gas as ideal, derive its compressibility c_g as funtion of pressure P.

(0.3 points)

4 Fractured Reservoir

Most of the world's largest oil reservoirs have a different structure, which is not a pile of small balls with fluid between, but a very complex system of porous medium and fractures as shown in Figure 9 (left). Fortunately, such reservoirs could be easily modeled with a stack of sugar cubes, as in Figure 9 (right).

In the model supposed that the production from fractured reservoir goes from the matrix to the fracture and therefrom to the well. Thus, matrix does not produce directly into the well. Such a simple model gives incredibly good results for oil production forecasts with an equation:

$$q = \sigma \frac{a^3}{\mu}(P_m - P_f) \ , \tag{8}$$

where q is a flow rate from matrix to fracture, P_m is average pressure at the matrix, P_f is pressure at fracture, near the boundary of the sugar cube with a side a, and σ is shape factor, related to the dimensions of the sugar cubes.

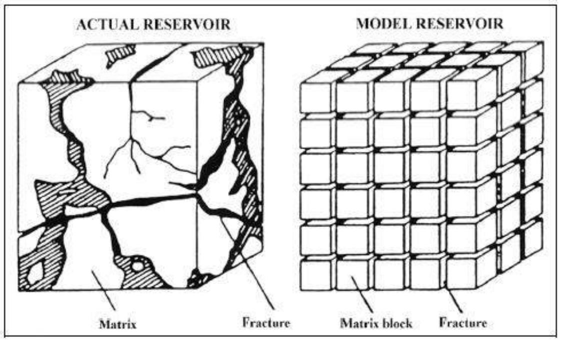

Figure 9: Cubic stacking of identical spherical grains

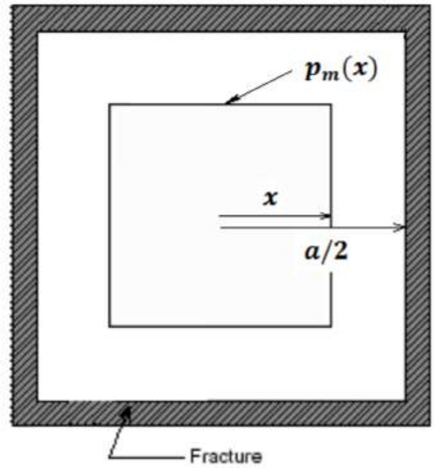

Figure 10: Cubic stacking of identical spherical grains

The goal of this part of the problem is to estimate shape factor σ. Consider a cube with the side a filled with a porous medium with porosity φ, permeability k, and compressibility c_r. Oil flows with a constant rate q symetrically from the center of the cube to its boundaries, where pressure is equal P_f, which changes with time t. Furthermore, if the well producing at constant flow rate then the cell pressure will decline in such a way that

$$\frac{dP_m}{dt} \approx \text{ const for all } x \text{ and } t. \tag{9}$$

(a) Calculate pressure distribution inside the cube $P_m(x)$ in terms of P_f, a, μ, k, and, q.

(2.2 points)

(b) What is the shape factor for the cube with side a ? (0.5 points)