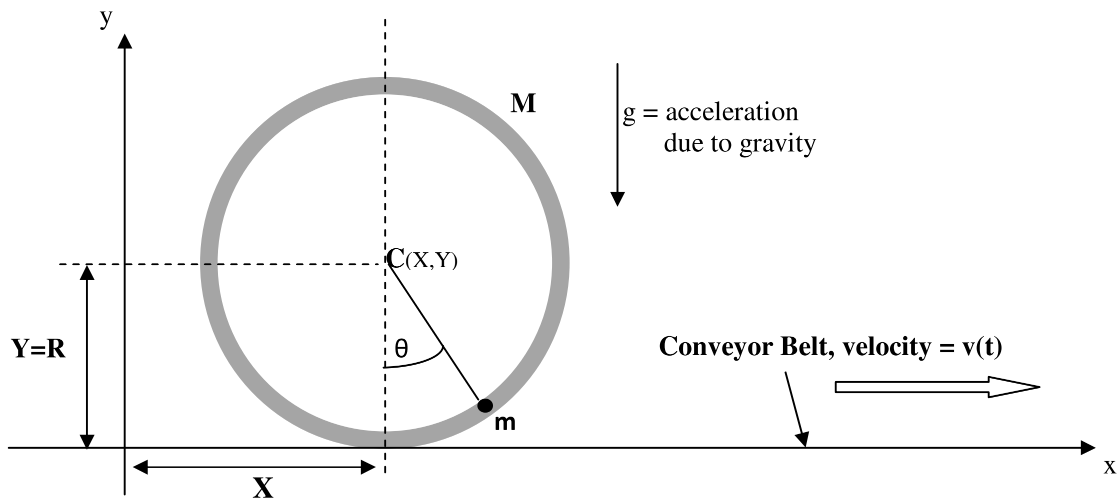
Deadline March, 31 2012

A loaded hoop consists of a 'load' or point mass m, embedded in a rigid hoop with mass M and radius R. The hoop is on a flat horizontal conveyor belt. At time t, the load subtends an angle θ with the vertical while the horizontal and vertical coordinates of the center C of the hoop are X and Y respectively.

In this problem assume that the hoop does not tilt and is always in the same vertical plane. The gravitational acceleration is g.

Use the following conventions for the derivatives of any function f : $\dot{f} \equiv \frac{df}{dt}$ **and** $f' \equiv \frac{df}{d\theta}$



Part 1. Preliminary Calculations

For Part 1, assume that the hoop is always in contact with the conveyor belt and that X and θ are independent (i.e. do not assume any constraint such as a non-slip condition etc.).

1.A. *Acceleration*: Express the horizontal and vertical components, a_x and a_y, of the acceleration of the load in the laboratory frame interms of R, $\ddot{X}$, θ, $\dot{\theta}$, and $\ddot{\theta}$.

1.B. *Energy*: Use the convention that the gravitational potential energy of the loaded hoop is zero when the load is at the bottom. Express the total mechanical energy, E, of the loaded hoop in terms of M, m, g, R, θ, $\dot{\theta}$, and $\dot{X}$.

Part 2. Oscillation on a Stationary Conveyor Belt

For Part 2 assume that the conveyor belt is at rest. If the loaded hoop is given a small angular displacement from the vertical and released from rest the system will oscillate.

2.A. *Period when friction is negligible*: Assuming that friction is negligible, determine the period of the small oscillation, T_A of the loaded hoop.

2.B. *Period when the hoop does not slip*: Assuming that friction is high enough to ensure that there is no sliding, determine the period of small oscillation, T_B of the loaded hoop.

Part 3 Stable Angular Orientations on an Accelerating Conveyor Belt

For Part 3, assume the following:

- The loaded hoop is placed on the conveyor belt at time $t = 0$ and initially $\dot{\theta} = \dot{X} = X = 0$.
- The velocity at any point of the belt is given by $v = Kt$ where K is a proportionality constant.

If the loaded hoop is placed on the conveyor belt at $t = 0$ with carefully chosen initial values of θ, then it will maintain a stable angular orientation provided K/g, μ_s, μ_k, and m/M satisfy appropriate conditions – assume that these conditions are satisfied only in Part 3.

3.A. Determine the stable orientation angle $\theta = \alpha_1$ as a function of m/M when $m/M \geq 7$, $\mu_k = \sqrt{3}/3$, $K = \sqrt{3}g$ and $\mu_s = 1.0$.

3.B. Determine the stable orientation angle $\theta = \alpha_2$ as a function of m/M when $m/M \geq 7$, $\mu_k = \sqrt{3}/3$, $K = \sqrt{3}g$ and $\mu_s = 2.0$.

Part 4. Oscillation, Rotation, and Sliding on an Accelerating Conveyor Belt

For Part 4, assume the following:

- $M = m$
- The loaded hoop is placed on the conveyor belt at time $t = 0$ and initially $\dot{\theta} = \dot{X} = \theta = X = 0$.
- The velocity at any point of the belt is given by $v = Kt$ where K is a positive constant.
- Let $\omega \equiv \frac{d}{dt}\theta$.

4.A. When the coefficient of static friction is extremely high (as will be assumed here), two types of motion may ensue-
Type A: The loaded hoop undergoes angular oscillations forever and never slides.
Type B: The loaded hoop completes at least one rotation without sliding.

(4.A.1) *Type A Motion.* For Part 4.A.1, suppose that the loaded hoop is observed to undergo Type A Motion and maximal angular displacement of the load is observed to be $\theta_{maximum} \equiv \beta = 30°$. Deduce an algebraic expression for K/g in terms of β and determine its numerical value.

(4.A.1) *Type B Motion.* When K/g exceeds a threshold value γ, the loaded hoop undergoes Type B motion. Determine the numerical value of γ to two significant figures.

4.B. *Sliding Motion.* Now assume that the value of K/g is the same as the value that you were asked to calculate in Part 4.A.1, but this time do not assume that the coefficient of static friction is extremely high. When the coefficient of static friction is less than a threshold value $\mu_{s,0}$, the loaded hoop will not undergo Type A motion because it will slide. Determine the numerical value of $\mu_{s,0}$ to two significant figures.

Hint:

1. $\dfrac{d^2}{dt^2}\theta = \omega \dfrac{d\omega}{d\theta}$