Problem 4 Heat capacity of moist porous solid
Deadline April, 30 2012

When fabricated or in use, solid porous bodies may be exposed to moisture penetrating their pores in a form of difficult–to–remove water. In this problem, you are asked to investigate the moisture content influence on thermal properties of porous solids.

The porous solid considered in the problem contains only closed pores with a little amount of water in them (Fig. 1). A volume of liquid water is negligible compared to that of the pore. The water is in equilibrium with the vapour phase. For simplicity the pores are taken to be void of any other gases. The pore heat expansion also can be neglected.

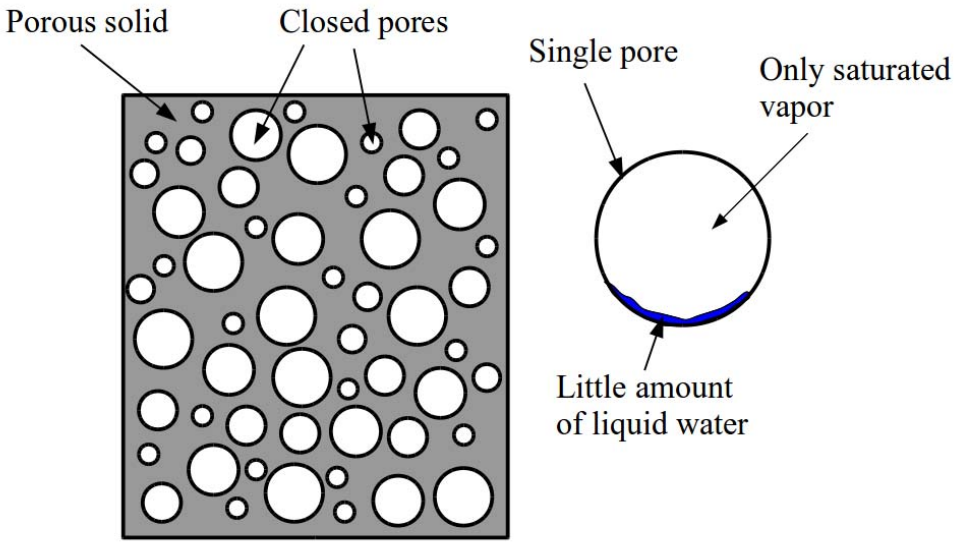

Figure 1: Porous solid and single pore.

Part 1. Pore contents

To begin with, consider processes involving a liquid and vapour in a single pore. Let an amount of liquid water with mass m_{W0} be in the pore of the volume V . The saturated vapour pressure is p_0 , temperature is T_0 . The molar mass of water is M , specific heat capacity is c_W , specific heat of evaporation is L. The vapour molar heat capacity at constant volume is $3R$. Assume that the specific heat capacity, specific heat of evaporation, and vapour molar heat capacity do not depend on temperature and pressure.

1.A. Find the mass of vapour m_{S0} in the pore.

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On heating a fraction of water evaporates and the vapour content in the pore increases. The saturated vapour pressure depends on temperature as follows:

$$p = p_0 e^{\frac{ML}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)}, \tag{1}$$

where p_0 is the pressure at temperature T_0.

The saturated vapour pressure variation near a curved surface of water can be neglected.

1.B. How do the masses of vapour and liquid water depend on temperature ($m_W(T)$ and $m_S(T)$)?

Let the vapour mass be equal m_S at a certain temperature T . The temperature of pore content increases from T to $T + \Delta T$.

1.C. Considering the temperature change to be small ($\Delta T/T \ll 1$), write down the expression for the vapour mass change ($\Delta m_S(T)$).

Hint. use the following approximations valid for $x \ll 1$:

$$e^x \approx 1 + x \qquad \text{and } (1+x)^\alpha \approx 1 + \alpha x. \tag{2}$$

1.D. How does the heat capacity $C_P(T)$ of the pore contents depend on temperature. Assume that the water does not evaporate completely.

At temperature $T_0 = 3.0 \times 10^2\,\mathrm{K}$, the saturated vapour pressure is $P_0 = 3.5\,\mathrm{kPa}$ and liquid water occupies $\delta = 0.3\,\%$ of the pore volume. The water molar mass is $M = 18 \times 10^{-3}\,\mathrm{kg/mole}$, the density is $\rho_w = 9.5 \times 10^2\,\mathrm{kg/m^3}$, the specific heat capacity is $c_w = 4.2 \times 10^3\,\mathrm{J/K.mole}$ and the specific heat of evaporation is $L = 2.3 \times 10^6\,\mathrm{J/kg}$. Universal gas constant is $R = 8.3\,\mathrm{J/K.mole}$. Assume that the density does not depend on temperature and pressure.

1.E. At what temperature T_1 will water completely evaporate? You can calculate the value of the temperature numerically.

1.F. Estimate the values of all terms in the expression for heat capacity of pore content $C_P(T)$. Write down the expression for $C_P(T)$ retaining only two the most significant summands. What is the physical meaning of these terms?

Let's define the specific heat capacity of the pore contents as a ratio of heat capacity of the pore contents to the mass of the pore contents:

$$c_p = \frac{C_p}{m_W + m_S} \tag{3}$$

1.G. Calculate the values of the specific heat capacity for the pore contents at $T = T_0$, $T \to T_1$, and $T > T_1$.

Part 2. Porous solid

In this part, you should solve the problem by taking into account properties of the porous body.

Let the porosity (a ratio of the volume of pores to the total body volume) be $\xi = 2/3$. The density of a body skeletal portion is $\rho_S = 6.0 \times 10^2\,\mathrm{kg/m^3}$ and its specific heat capacity is $C_S = 4.0 \times 10^2\,\mathrm{J/kg.K}$. Initial conditions for the pore contents is the same as in part 1: $\delta = 0.30\,\%$, $T_0 = 3.0 \times 10^2\,\mathrm{K}$, and $p_0 = 3.5\,\mathrm{kPa}$.

2.A. Write down the expression for the specific heat capacity $c(T)$ of the moist porous solid.

2.B. Find numerical values of the specific heat capacity of porous solid at $T = T_0$, $T \to T_1$ and $T > T_1$.

2.C. Plot the specific heat capacity versus temperature for $T \in (T_0; T > T_1)$.