Deadline May, 31 2012

Constants:
Gravitational acceleration : $g = 9.8\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$
Water density: $\rho = 1000\,{}^{\mathrm{kg}}/_{\mathrm{m}^3}$

Tsunami is an enormous ocean wave phenomena generated by sudden water displacement due to oceanic seismic (earthquake) activity. Tsunami is extremely devastating to human population living in the coastal region. Recent examples are the Indian Ocean tsunami (2004) and Tohoku-Japan tsunami (2011) that have claimed many as many as 230,000 and 15,000 lives respectively .

In this problem we will explore the basic physics of Tsunami that will help us to appreciate some of its fundamental characteristics and help disseminate some potentially life-saving knowledge in the event of such calamity. In this problem we will use some estimated data of the 2004 Indian Ocean tsunami produced by an earthquake off the coast of Sumatra, Indonesia as shown below..

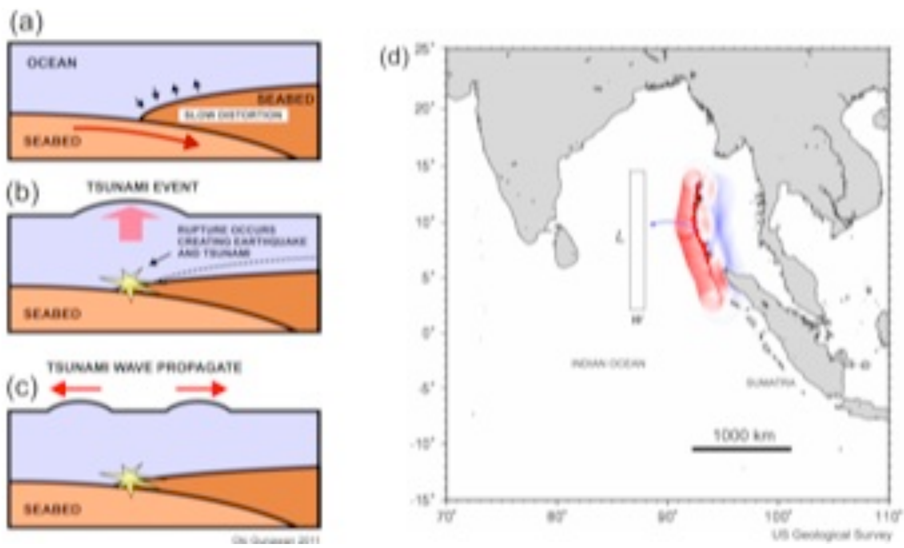

Figure 1: (a-c) The sequence of a Tsunami event. (d) The wave structure of the 2004 Indian Ocean Tsunami. Red wave travels to the west and blue to east. *Dotted box*: Effective area $L \times W$ for the initial tsunami wave generated along the fault line (See Question 1).

Part 1. Energy of tsunami

The ocean floor seismic rupture suddenly displaces a large body of water above it along the fault line as shown in Figure 1. *This excess body of water* will be dissipated as tsunami waves that mainly propagates to the left and right. Let us make a simple model of the excess body of water with *triangular cross section* as shown in Figure 2. The initial water displacement is $h = 5\,\text{m}$ covering a large area of fault line of $L = 1400\,\text{km}$ and width $W = 150\,\text{km}$. (Note that h is really in meter, very small compared to L and W).

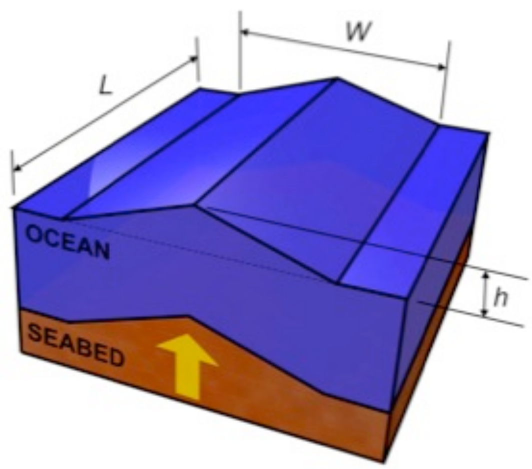

Figure 2: A model of the initial displacement of a tsunami wave

1.A. Calculate the excess energy that will be dissipated as tsunami ! Assume that this excess body of water is at rest right after the seismic event.

1.B. Compare this energy with Hiroshima atomic bomb that equals to 12500 tons of TNT explosion. One ton of TNT explosion yields 4.2×10^9 J of energy.

Part 2. Speed of tsunami

Tsunami wave, in the open ocean, has small amplitude ($a \sim 5\,\text{m}$), extremely long wavelength ($\lambda \sim 100\,\text{km}$) and traveling on a very deep ocean ($d \sim 5\,\text{km}$). Thus, surprisingly, tsunami can be considered as "shallow water waves" where $a \ll d \ll \lambda$. Here the speed of tsunami (or phase velocity) is given as:

$$v = \sqrt{gd} \tag{1}$$

Let us make a very simple derivation of the tsunami speed by using a simple model of one half tsunami wave using a water tank model as shown below. The water is tilting back and forth from left to right given a slight initial imbalance of height. Thus the height a will oscillate with time. Let us assume that the width of the water tank is half the wavelength of the tsunami wave λ. The length of water tank is L. Note: For tsunami (shallow water) wave, we assume: $a \ll d \ll \lambda$.

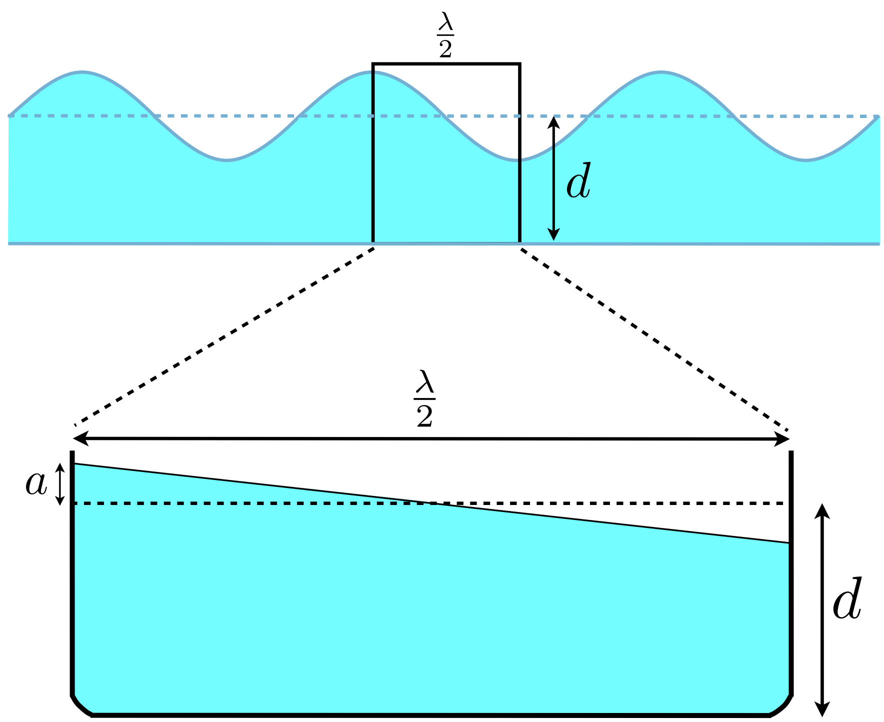

Figure 3: Water tank model of a tsunami wave to estimate its wave velocity

2.A. Write down the horizontal horizontal velocity of the water element as a function of horizontal position x, a and/or its derivative. Hint: the velocity at the edge of the water tank is zero.

2.B. Express the total kinetic and potential energy of the system in terms of a and/or its derivative!

2.C. Show that the system exhibits a simple harmonic oscillator. Calculate the period of oscillation T of the water!

2.D. The tsunami displaces wave with wavelength λ in a time period T. The wave speed or "phase velocity" is given as: $v = \lambda/T$. Show that: $v \propto \sqrt{gd}$ ($\propto$ means proportional to).

Part 3. Tsunami warning system

Use the tsunami speed equation as given in Eq. 1 above.

3.A. Calculate the average tsunami speed and period in the Indian ocean with depth $d = 4000\,\text{m}$ and a long wavelength of $\lambda = 100\,\text{km}$. What object that travels close to that speed? (bike, car, passenger jet plane?) .

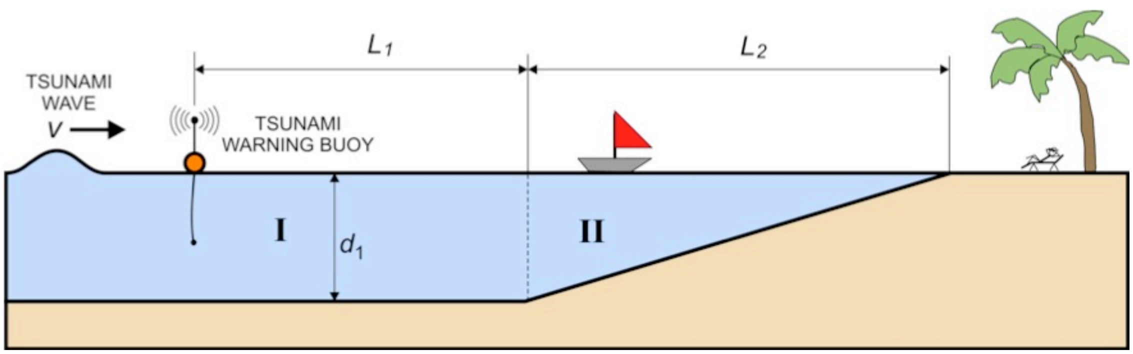

Figure 4: A tsunami warning system in a coastal area.

3.B. A tsunami warning system is installed near a coastal region with the seafloor profile that can be modeled as shown in Figure 4 : a constant depth followed by a linearly decreasing depth region. We have a tsunami warning buoy that can detect a tsunami wave front at $t = 0$. Calculate the time Δt when the tsunami will hit the land!

Part 4. Tsunami characteristics on land impact

In the ocean where the seafloor depth is constant (region I) The tsunami waves has a characteristics height h and coming to the land with linearly increasing ocean floor (region II).

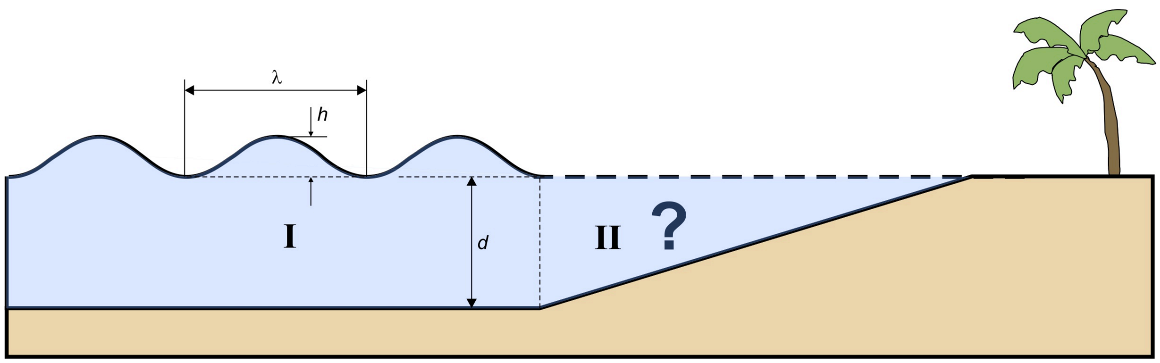

Figure 5: Tsunami characteristics on land impact.

4.A. Investigate what happens to the tsunami wave height h as it comes crashing to land. Express the relationship of h as a function of depth d. **Hint**: The period of the tsunami wave is constant everywhere.

4.B. Sketch the tsunami wave as it approaches the land.

4.C. Estimate the height of tsunami wave crashing to Aceh coast (Sumatra, Indonesia) with final depth of 2 m (before the wave breaks) coming from Indian ocean with initial wave height of 5 m and depth of 4000 m. Comment on your finding to explain the devastating effect of tsunami.

Part 5. Tsunami Drawback effect

An early warning sign of the incoming tsunami is the "drawback effect" where the sea water near the beach recedes significantly for hundreds of meter and for about half the tsunami period (about few minutes). An observer may be attracted to the exposed beach unaware of the oncoming danger. (In the 2004 Indian Ocean Tsunami event a little tourist boy in Thailand saved the lives of many because he learnt about this effect from school.)

Let us explore this drawback effect by the following model. Assume that we put a test buoy B on the water surface that will track the water particle there. Note that there are two movements: First is the cyclic up and down movement with a period $T = v_0/\lambda$ where v_0 is the phase velocity of the wave, and second : the horizontal movement because the buoy is dragged by the traveling wave, but *with the speed less* than the wave phase velocity v0.

Thus to track the trajectory of buoy B we can use a Yo-Yo model who rotates with period T but the center moves with speed less than v_0.

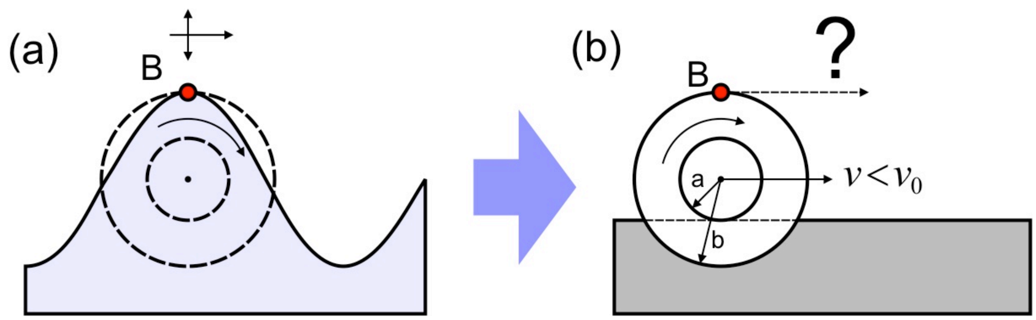

Figure 6: (a) A test buoy model and (b) A yo-yo model to investigate the "tsunami drawback effect"

5.A. Sketch the trajectory of the buoy B as a function of distance as the "Yo-Yo" rolls.

5.B. Highlight your observation that explains the tsunami "drawback" effect !