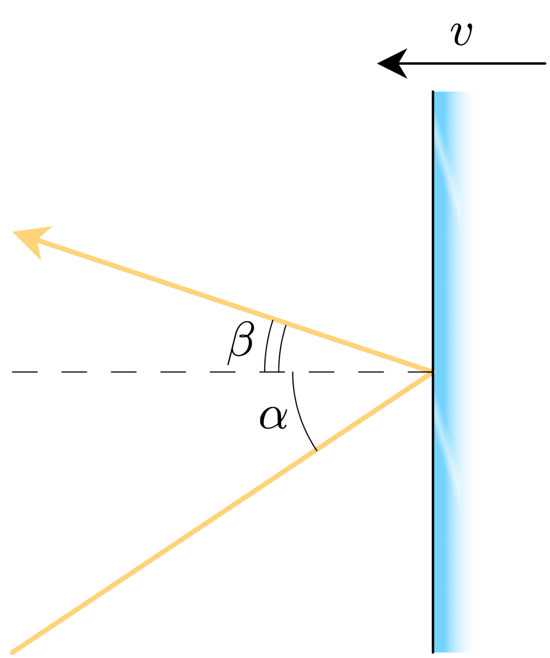
As the old proverb goes, "seeing is believing", and since seeing depends on light, what we see is highly dependent on the nature of light. These following problems aim to investigate the peculiarities of vision in the light of Relativity theory, particularly those arising from the effect of lights nonzero travel time.

Part 1. Cameras

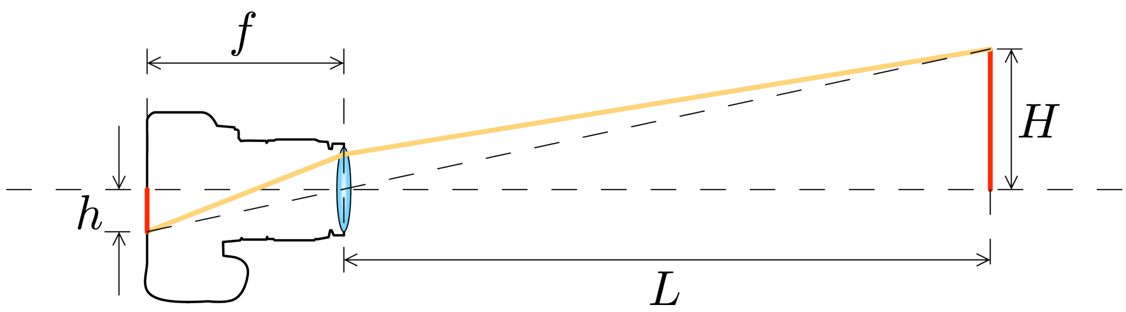
1.A. A perfectly reflecting mirror moves at a relativistic speed v in a direction normal to the plane of its surface. At a certain moment, a narrow beam of light strikes the mirror at an angle of incidence α. Determine the angle of reflection β of the beam, and show that in the limit of small angles, the ratio of α and β is given by

$$\frac{\alpha}{\beta} = \frac{c+v}{c-v} \tag{1}$$



1.B. Consider a simplified view of image formation in a camera, wherein said device works by capturing light from an object and projecting it onto a screen. If the object is far away, light from it would obviously be focused at the focal point of the cameras lens.

It is easily shown that the size of the image formed on the screen of the camera, for an object of size H at a distance L from the camera, is given by $h = Hf/L$.

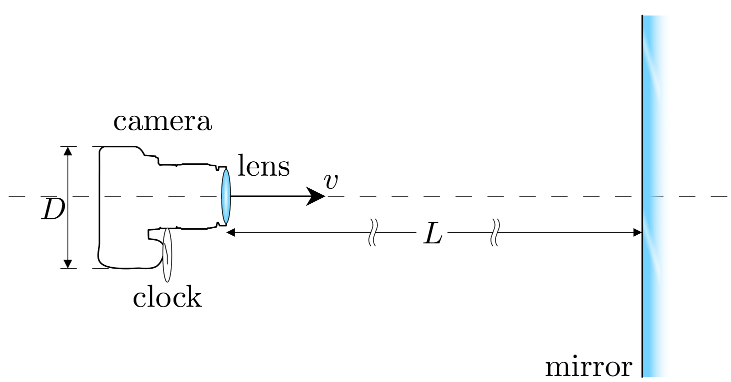


Suppose now that the camera is approaching the object at a relativistic speed v while rapidly taking a photograph; assume the distance between the camera and the object is still L.

(a) How will the image on the moving camera screen look like (bigger or smaller) compared to the original case where the camera is at rest?

(b) At what rate will the height of the image on the screen be changing at this instant?

1.C. A camera with video-recording capability, width D, lens focal length f and a clock attached in front of it is moving relativistically with speed v towards a perfectly reflecting plane mirror as shown in the figure below.



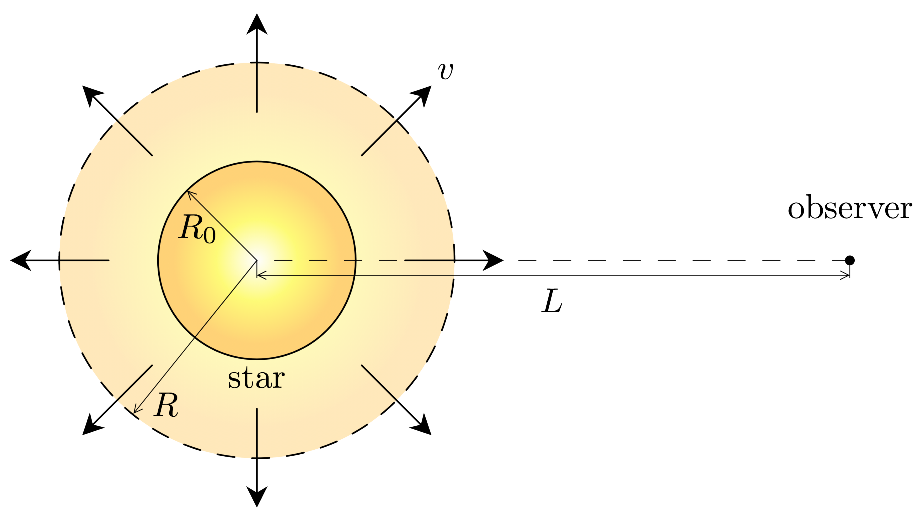
(a) At a certain moment, the camera is located at a distance L from the mirror in the mirrors rest frame. Determine the rate of change of the camera images width as recorded by the camera itself, given that $L \gg d$.

(b) At that time an observer moving together with the camera notes that the difference between the reading of the cameras clock and the images clock is A seconds and that the ticking speed of one clock is B times faster than the other. Using this information, find the distance between the camera and the mirror as seen in the rest frame of the mirror, given that $A > 0$ and $B > 1$.

(c) Show that at the moment the camera reaches the mirror, both its clock and its images will show the same time.

Part 2. Expanding Stars

A perfectly spherical star of spectral class G initially has a radius R_0. All of a sudden, a cataclysmic event at $t = 0$ in the stars rest frame causes it to start expanding at a constant relativistic rate $dR/dt = v$. An observer on Earth, at a distance $L \gg R$ from the star, is observing this strange phenomenon. The Earth frame can be treated as an inertial frame.



2.A. Determine the apparent angular size of the star as a function of time, $\theta(t)$, as seen by the observer. Hint: it may be a good idea to plot this function.

2.B. When the star reaches a certain radius R_m, its light suddenly dies out. Describe, both qualitatively and quantitatively, the shape of the star as seen by the observer, from the moment the stars radius reaches R_m, and onwards to eternity.

Assume the star is a perfect blackbody so that light from the stars interior cannot reach the observer on Earth.