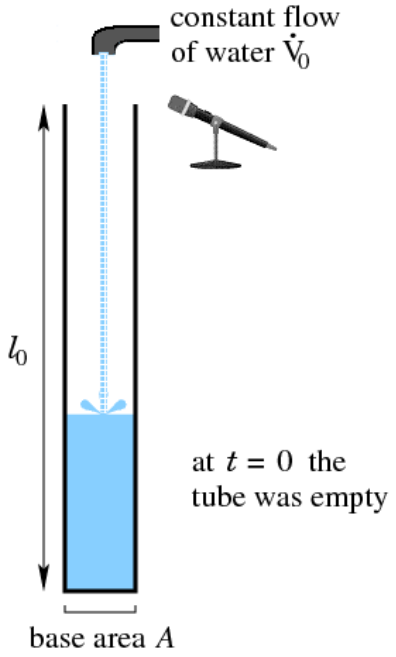
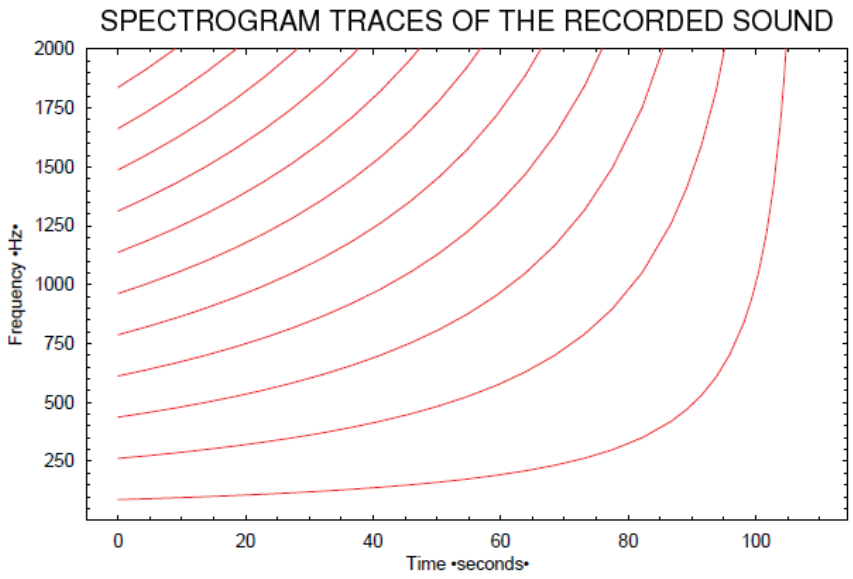
Problem 9 Pouring Water into a Tube
Deadline July, 31 2012

Part 1.

A steady stream of water from a pipe valve slowly fills up a relatively slender cylindrical tube of height $l_0 = 1\,\mathrm{m}$ and base area $A = 1.26 \times 10^{-3}\,\mathrm{m}^2$. As the tube is being filled up with water, a microphone records the sound that is being produced. The spectrogram of the recorded sound signal reveals the time evolution of the prominent audible frequencies as the water level in the tube rises. The details are illustrated in the following schematics:





Based on the available information of the setup and the data given, determine reasonably reliable numerical estimates for the following quantities:

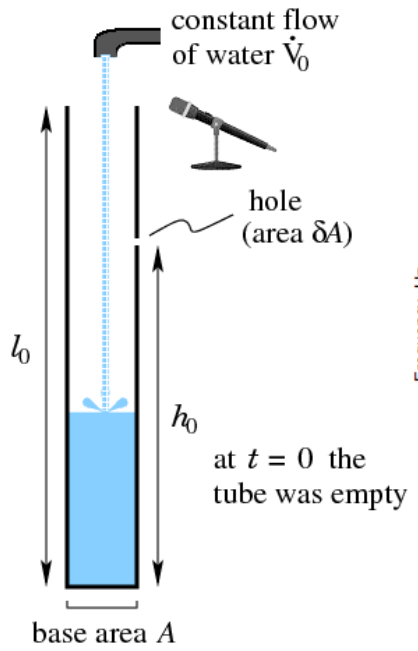
1.A. the sound speed c_s under local atmospheric conditions (unit: m/s)

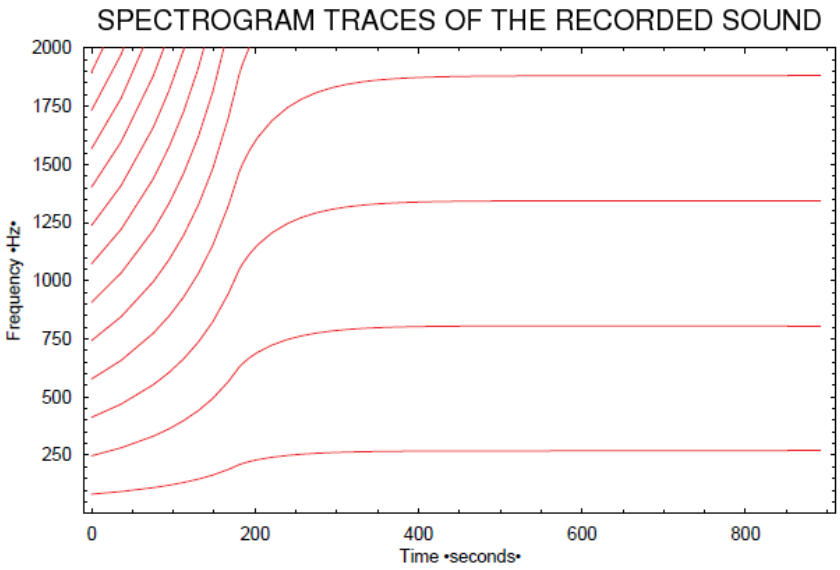
1.B. the flow rate of the water stream $\dot{V}_0$ into the tube (unit: m^3/s)

For the purpose of aiding analysis and calculations, a larger version of the above spectrogram plot is provided on page 4 of this exam paper.

Part 2.

On another occasion, similar experiment is performed under slightly different conditions and settings. A steady stream of water slowly fills up a relatively slender cylindrical tube of height $l_0 = 1\,\mathrm{m}$ and base area $A = 1.26 \times 10^{-3}\,\mathrm{m}^2$, but this time the side of the tube has a small hole at a certain location. Again, a microphone records the sound that is being produced as the tube is being filled up with water. The gravitational acceleration is $g = 9.8\,\mathrm{m/s}^2$. The settings and the resulting spectrogram traces are shown in the schematics below:





Based on the available information of the setup and the data given, determine reasonably reliable numerical estimates for the following quantities:

2.A. the sound speed c_s under local atmospheric conditions (unit: m/s)

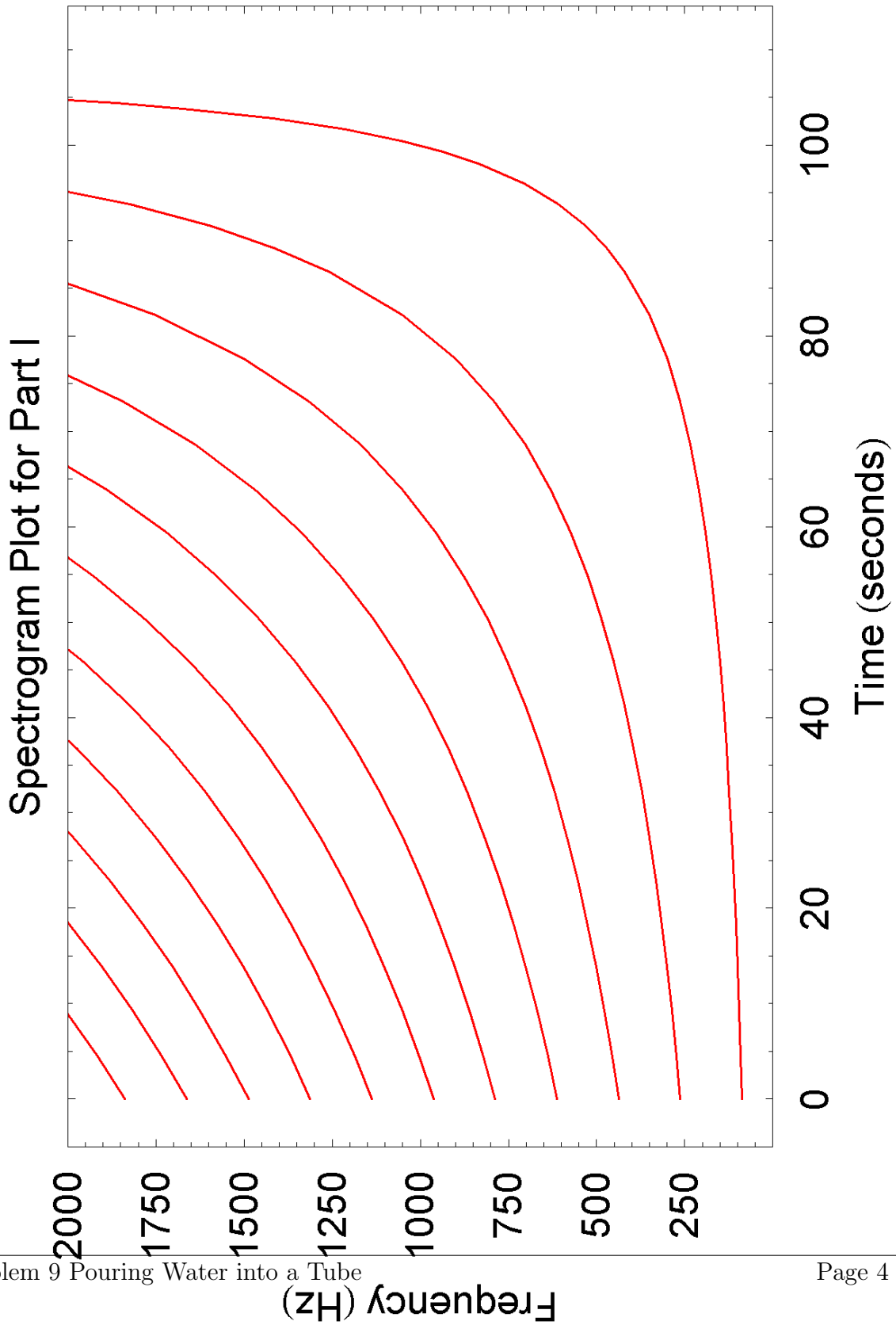
2.B. the flow rate of the water stream $\dot{V}_0$ into the tube (unit: m^3/s)

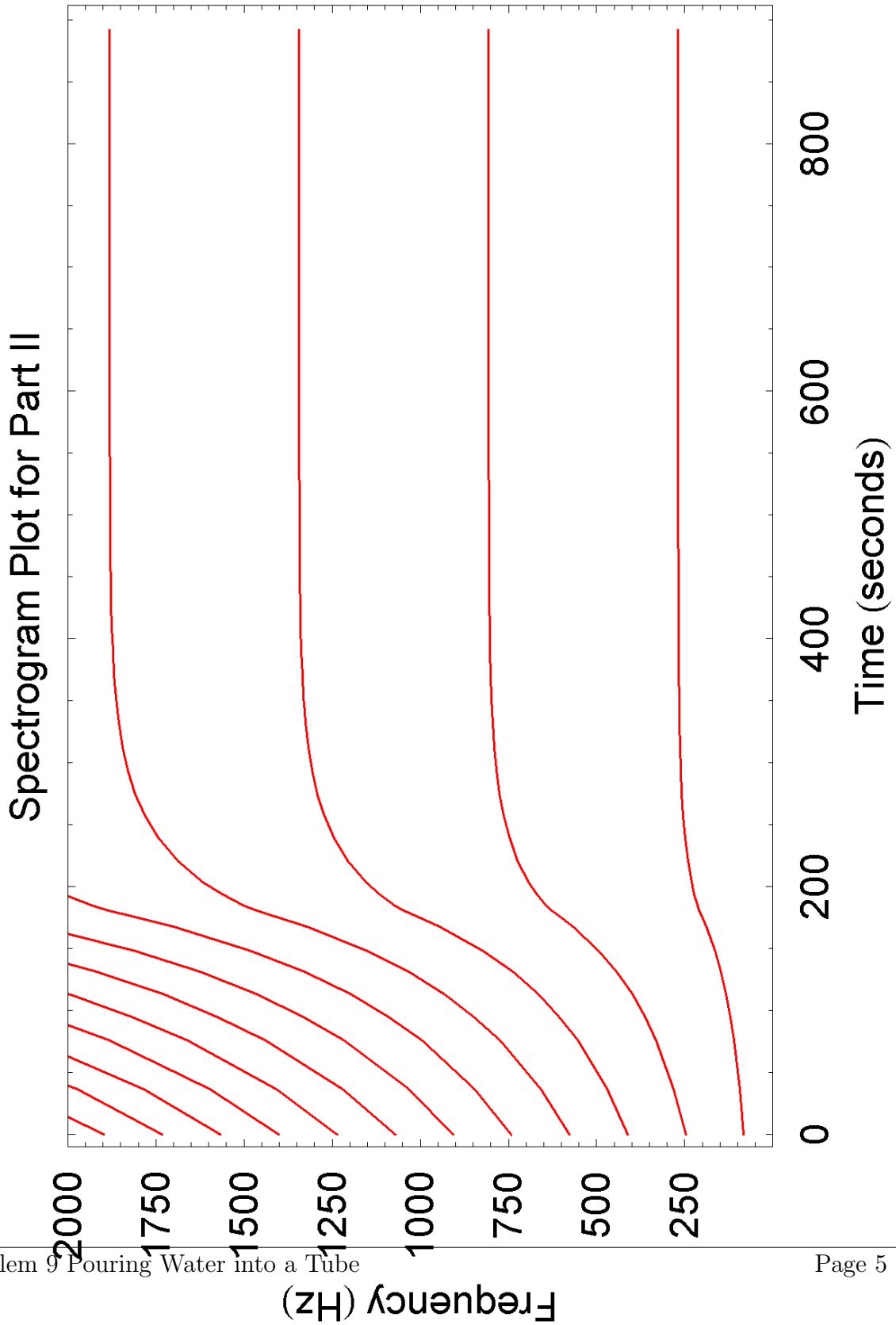
2.C. the height h_0 where the hole is located (unit: m)

2.D. the area δA of the hole (unit: m^2)

For the purpose of aiding analysis and calculations, a larger version of the above spectrogram plot is provided on page 5 of this exam paper.

Note: Ignore the effect of vena contracta (i.e. diameter contraction) in the jet of water leaking from the hole when the water level is above the hole position.


Spectrogram Plot for Part I
Time (seconds)
0
20
40
60
80
100
Frequency (Hz)
2000
1750
1500
1250
1000
750
500
250


Spectrogram Plot for Part II
Time (seconds)
0
200
400
600
800
Frequency (Hz)
2000
1750
1500
1250
1000
750
500
250