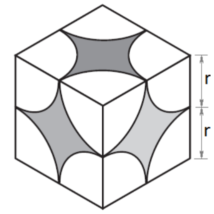
Physics of the Oil and Gas Production

SOLUTION



1.1 Let's cut a cube from the pile of balls, as shown in figure on the right. Using definition of porosity:

$$\varphi = \frac{(2r)^3 - 8\cdot\frac{1}{8}\left(\frac{4}{3}\pi r^3\right)}{(2r)^3} = 1 - \frac{\pi}{6}$$

$$\boxed{\varphi = 0.476 \ \ or \ \ 47.6\%}$$

1.2 If fluid is incompressible, fluid velocity along the tube is constant. It varies in the radial direction only, what is due to viscous forces. Let's consider a part of the fluid with a cylindrical shape, with radius y, which is coaxial to the cylinder with radius r_0. Using definition of the internal friction, force balance could be written in the following way:

$$(p_1 - p_2)\cdot \pi y^2 = -2\pi y L_0 \mu \frac{dv}{dy} \qquad (1.2.1)$$

After rearranging variables

$$\frac{(p_1 - p_2)}{2\mu L_0}\int_{r_0}^{y} y dy = -\int_0^v dv$$

which results in velocity distribution:

$$\boxed{v(y) = \frac{(p_1 - p_2)}{4\mu L_0}(r_0^2 - y^2)} \qquad (1.2.2)$$

1.3 Total fluid volume, flowing through the tube in a unit of time:

$$q = \int_0^{r_0} v * 2\pi y dy = \frac{(p_1 - p_2)\pi}{2\mu L_0}\int_0^{r_0}(r_0^2 y - y^3)dy = \frac{(p_1 - p_2)\pi r_0^4}{8\mu L_0} \qquad (1.3.1)$$

Comparing with Poiseuille equation

$$\boxed{k_0 = \frac{r_0^2}{8}} \qquad (1.3.2)$$

1.4 For estimations one can assume that porous medium could be modeled as tubes with the radiuses equal to the size of balls:

$$k \approx \frac{r_0^2}{8}\left(1 - \frac{\pi}{6}\right)^2 \qquad (1.4.1)$$

$$\boxed{k \approx 3\cdot 10^{-14}\ m^2}$$

1.5 From law of conservation of mass and the condition that fluid is incompressible can be concluded that flow rate is constant everywhere:

$$q = \frac{k_1}{\mu} A \frac{P_{in} - P_b}{L_1} = \frac{k_2}{\mu} A \frac{P_b - P_{out}}{L_2}$$

$$\boxed{P_b = \frac{\frac{k_1}{L_1}P_{in} + \frac{k_2}{L_2}P_{out}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}} \qquad (1.5.1)$$

1.6 Using Eq. (1.5.1)

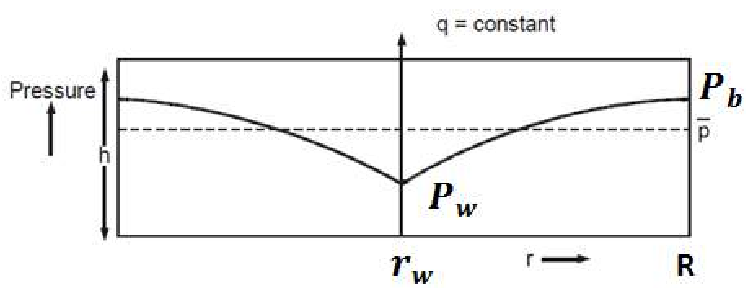
$$q = \frac{k_{eff}}{\mu} A \frac{P_{in} - P_{out}}{L_1 + L_2} = \frac{A}{\mu}\frac{k_1}{L_1}\left[P_{in} - \frac{\frac{k_1}{L_1}P_{in} + \frac{k_2}{L_2}P_{out}}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}\right]$$

$$\boxed{k_{eff} = \frac{k_1 k_2 (L_1 + L_2)}{k_1 L_2 + k_2 L_1}} \qquad (1.6.1)$$

2.1
$$v_w = \frac{q}{\pi r_w^2} = \frac{30/86400}{3.14\cdot 0.1^2} = 1.1\cdot 10^{-2}\,\frac{m}{sec}$$
$$v_{res} = \frac{q}{2\pi r_w h} = \frac{30/86400}{2\cdot 3.14\cdot 0.1\cdot 10} = 5.5\cdot 10^{-5}\,\frac{m}{sec}$$
Velocity at the reservoir is very small, however it depends on the definition of the fluid velocity in the reservoir. Actually fluid particles moves a few orders of magnitude faster than Darcy's velocity $\approx \frac{v_{res}}{\varphi}$.

2.2 For incompressible fluid
$$q = \frac{k}{\mu} 2\pi r h \frac{dP}{dr} = const \qquad (2.2.1)$$
Rearranging variables
$$\int_{P_w}^{P} dP = \frac{q\mu}{2\pi k h}\int_{r_w}^{r} \frac{dr}{r}$$
$$P - P_w = \frac{q\mu}{2\pi k h} ln\left(\frac{r}{r_w}\right) \qquad (2.2.2)$$
Finally
$$\boxed{P_b - P_w = \frac{q\mu}{2\pi k h} ln\left(\frac{R}{r_w}\right)} \qquad (2.2.3)$$

2.3



3.1
$$q = \frac{dV_{fluid}}{dt} = \frac{d(\varphi V_{res})}{dt} = \varphi V_{res}\left(\frac{1}{V_{res}}\frac{dV_{res}}{d\bar{p}}\right)\frac{d\bar{p}}{dt} = -2\varphi L^2 h c_r \frac{d\bar{p}}{dt}$$
$$\boxed{\alpha = -2\varphi L^2 h c_r} \qquad (3.1.1)$$

3.2 Applying Darcy's law:
$$q = 2\frac{k}{\mu} L h \frac{P_b - P_w}{L} = -2\varphi L^2 h c_r \frac{d\bar{p}}{dt} \qquad (3.2.1)$$
Average pressure can be estimated as:
$$\bar{p} \approx \frac{P_b + P_w}{2} \qquad (3.2.2)$$
Considering that P_w is constant:
$$d\bar{p} \approx \frac{1}{2} dP_b \qquad (3.2.3)$$
Solving Eq.(3.2.3) and (3.2.1)
$$\int_{P_b(0)}^{P_b(t)} \frac{dP_b}{P_b - P_w} = -\frac{2k}{\mu L^2 c_r \varphi}\int_0^t dt$$
$$ln\left(\frac{P_b(t) - P_w}{P_b(t) - P_w}\right) = -\frac{2k}{\mu L^2 c_r \varphi} t$$
Using definition of the flow rate from Eq. (3.2.1)
$$ln\left(\frac{q(t)}{q_0}\right) = -\frac{2k}{\mu L^2 c_r \varphi} t$$
$$\boxed{q(t) = q_o exp\left(-\frac{2k}{\mu L^2 c_r \varphi} t\right)} \qquad (3.2.4)$$

3.3

$$V_{produced} = \int_o^T q(t)dt \quad (3.3.1)$$

Using Eq. (3.2.4)

$$V_{produced} = -\frac{\mu L^2 c_r \varphi}{2k} q_o exp\left(-\frac{2k}{\mu L^2 c_r \varphi} t\right) |_o^T = \frac{\mu L^2 c_r \varphi}{2k} q_o \left[1 - exp\left(-\frac{2k}{\mu L^2 c_r \varphi} T\right)\right] \quad (3.3.2)$$

From the problem statement, volume of oil produced is a half of the initial reserves:

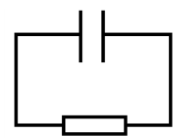
$$V_{produced} = \frac{1}{2} V_{res}\varphi = \frac{1}{2}(2L^2 h)\varphi \quad (3.3.3)$$

Thus

$$\boxed{T = \frac{\mu L^2 c_r \varphi}{2k} ln\left(\frac{1}{1-\frac{2kh}{\mu c_r q_0}}\right)} \quad (3.3.4)$$

Equation (3.3.4) gives rather bad results for real data, because the fluid flow model was very rough, nevertheless order of the magnitude usually is very close to the results from numerical simulations.

3.4 Equation for the flow rate $\boldsymbol{q(t)}$ derived in the part 3.2 is an exponential decline. Similar equation could be obtained for the circuit with a capacitor and a resistor. There a flow rate is analogous to the charge at the capacitor plate and term with compressibility is analogous to the resistor.



3.5 Ideal gas law:

$$V = \frac{\vartheta RT}{P} \quad (3.5.1)$$

Differentiating Eq. (3.5.1)

$$\left(\frac{dV}{dP}\right)_T = -\frac{1}{p^2}\vartheta RT \quad (3.5.2)$$

Using definition of the compressibility and Eq. (3.5.2) results in

$$\boxed{c_g = -\frac{1}{V}\left(\frac{dV}{dP}\right)_T = \frac{1}{p}} \quad (3.5.3)$$

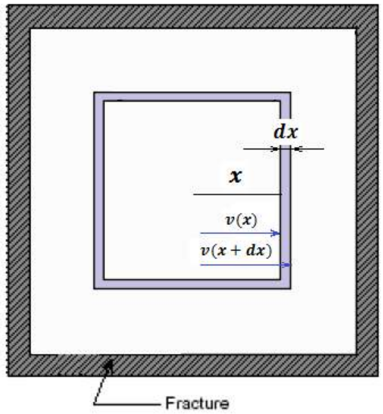
As it is seen with higher pressure gas behaves more like a fluid, with negligible compressibility. However during production, when pressure drops significantly, gas production deviates from exponential decline.

4.1 At first let's find the constant for pressure decline. The condition $\frac{dp_m}{dt} = const$ allows to write the following:

$$q = \frac{dV_{fluid}}{dt} = V_{fluid}\left(\frac{1}{V_{fluid}}\frac{dV_{fluid}}{dp_m}\right)\frac{dp_m}{dt} = -a^3\varphi c_r \frac{dp_m}{dt}$$

Or

$$\frac{dp_m}{dt} = -\frac{q}{a^3 \varphi c_r} = const \quad (4.1.1)$$



Material balance equation for the thin layer, shown in figure above:

$$v(x)A(x) - v(x+dx)A(x+dx) = \frac{dV}{dt} \tag{4.1.2}$$

A – stands for area; V – is volume of the analyzed thin layer with thickness dx. Using definition of compressibility, Eq. (4.1.2) can be simplified as

$$d(vA) = [6 \cdot (2x)^2 dx] \cdot c_r \varphi \frac{dp_m}{dt} \tag{4.1.3}$$

Using Eq. (4.1.1) and Darcy's law, Eq. (4.1.3) can be rewritten as

$$d\left(\frac{k}{\mu}\frac{dp_m}{dx} 6 \cdot (2x)^2\right) = -[6 \cdot (2x)^2 dx] \cdot \frac{q}{a^3} \tag{4.1.4}$$

$$\int d\left(x^2 \frac{dp_m}{dx}\right) = -\frac{\mu}{k}\frac{q}{a^3}\int x^2 dx$$

$$x^2 \frac{dp_m}{dx} = -\frac{\mu}{k}\frac{q}{a^3}\frac{x^3}{3} + C_1$$

where C_1 is some constant

$$\frac{dp_m}{dx} = -\frac{\mu}{k}\frac{q}{a^3}\frac{x}{3} + \frac{C_1}{x^2} \tag{4.1.5}$$

Eq. (4.1.5) will have meaning for the center of the cube ($x = 0$) only if $C_1 = 0$, otherwise pressure gradient $\frac{dp_m}{dx}$, which corresponds to the velocity in the center of the cube, will be infinite, what is meaningless from the point of physics. Thus,

$$\frac{dp_m}{dx} = -\frac{\mu}{3k}\frac{q}{a^3}x \tag{4.1.6}$$

Integrating one more time

$$\int_{p_f}^{p_m(x)} dp_m = -\frac{\mu}{3k}\frac{q}{a^3}\int_{a/2}^{x} x dx$$

$$\boxed{p_m(x) = p_f + \frac{\mu}{6k}\frac{q}{a^3}\left(\frac{a^2}{4} - x^2\right)} \tag{4.1.7}$$

4.2 Calculate average pressure in the matrix with using Eq.(4.1.7)

$$\bar{p}_m = \frac{\int_0^{a/2} p_m(x) \cdot 6 \cdot (2x)^2 dx}{a^3} = p_f + \frac{\mu q}{60ka} \tag{4.2.1}$$

Definition of the shape factor:

$$\sigma = \frac{q\mu}{ka^3(\bar{p}_m - p_f)} \tag{4.2.2}$$

Substituting Eq. (4.2.1) into (4.2.2) results in

$$\sigma = \frac{60}{a^2} \tag{4.2.3}$$