Two-Ball Body

Two uniform balls 1 and 2 of radii $R_1 = 2.00$ cm and $R_2 = 4.00$ cm respectively are made of the same material of the mass density $\rho = 1.50 \times 10^3\ \mathrm{kg/m^3}$. They are firmly glued together to form a rigid body as shown in Figure 1. In this problem you will have to investigate various kinds of motions of that rigid body called the two-ball body.

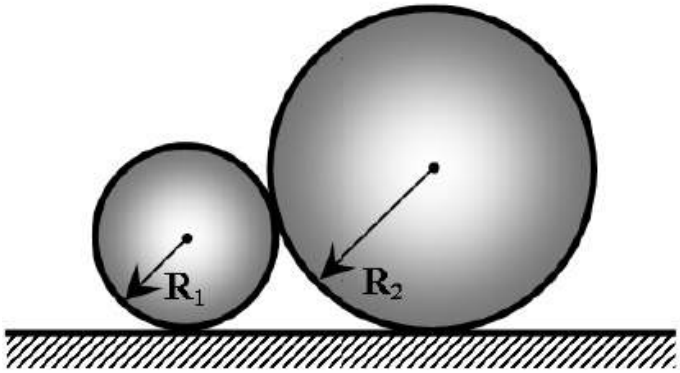

Figure 1: Two-ball body lying at rest on the flat horizontal surface.

Part A

A1 Find the distance x_0 from the center-of-mass of the two-ball body to the center of ball 1. 0.4 points

A2 Find the moment of inertia $I_\parallel$ of the two-ball body about the axis that passes through balls' centers. 0.6 points

A3 Find the moment of inertia $I_\perp$ of the two-ball body about the axis that passes through its center-of-mass and goes perpendicular to the line connecting the balls' centers. 1.0 points

Warning! At the solution of the following parts of this problem it is desirable to derive analytical answers to the questions. Use $r \equiv R_1/R_2$ to simplify the expression. However, if you find this difficult, you may calculate and provide numerical values only for each step of your solution.

Part B

Let the two-ball body be placed on the flat horizontal surface as shown in Figure 2. The smaller ball is right under the larger one such that the line connecting the centers of the two balls is strictly perpendicular to the surface. It is obvious that such an equilibrium position is unstable and an insignificant random deflection will set the two-ball body into a motion due to gravity whose acceleration is $g = 9.80\ \mathrm{m/s^2}$.

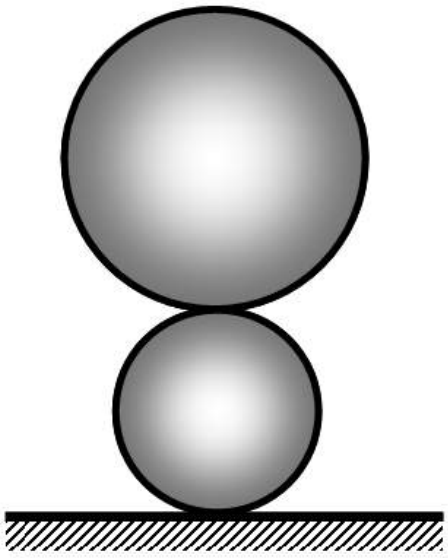

Figure 2: Initial position of the two-ball body for Part B.

B1 Assume that the friction between the lower ball and the surface is so strong that there is no slipping at all times. Find the velocities of balls' centers at the time moment right before the larger ball hits the ground. Draw a sketch with the depicted velocities of balls' centers. 1.5 points

B2 Assume now that there is no friction at all between the lower ball and the surface. Find the velocities of balls' centers at the time moment right before the larger ball hits the ground. Draw a sketch with the depicted velocities of balls' centers. 2.5 points

Part C

Let the two-ball body be placed on an inclined plane set at an angle $\alpha = 30^\circ$ against the horizontal. At the initial time moment the line connecting balls touching points with the surface is exactly parallel to the lower edge of the inclined plane as shown in Figure 3. In this Part assume that the friction between the balls and the surface is so strong that there is no slipping at all times. The two-ball body is released.

C1 Find the maximal velocities of balls' centers. 2.0 points

C2 Find the angular acceleration of the two-ball body when the velocities of balls' centers are maximal. 1.2 points

Now assume that the two-ball body is at rest on the inclined plane such that the line connecting balls' touching points with the surface is exactly perpendicular to the lower edge of the inclined plane as shown in Figure 4.

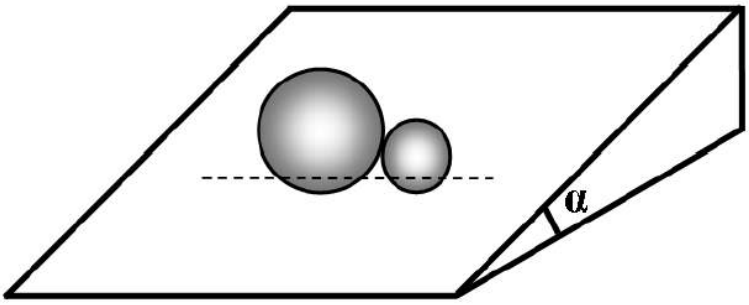

Figure 3: Initial position of the two-ball body for Parts C1-C2.

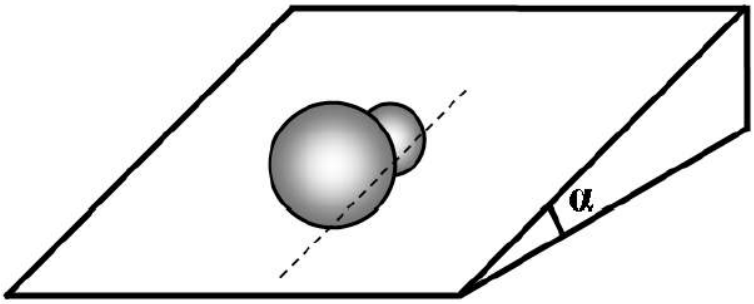

Figure 4: Initial equilibrium position of the two-ball body for Part C3.

C3 Find the angular frequency of small oscillations of the two-ball body around the equilibrium position shown in Figure 4. 0.8 points