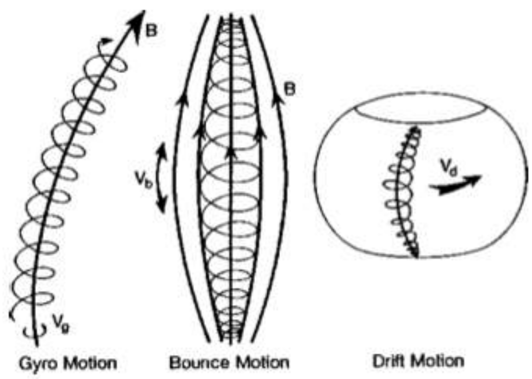
Particles in the magnetosphere

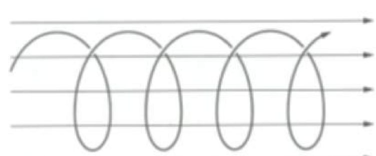
Deadline: April 3rd, 2013

Earth is a very interesting magnetic system. Earth is frequently approximated as a huge magnetic dipole and therefore its magnetic field is not uniform. Due to this non-uniformity there are some zones in the magnetosphere in which charged particles get trapped. These zones are known as Van Allen belts and particles inside them have three main movements: gyration around each magnetic field line (Gyro motion), movement along the field line (Bounce motion), and rotation of lines around the magnetic axis of the Earth (ignored in this question).



Due to the first two movements, particles travel along a helical path around the field lines. A key parameter to define such movement is the pitch angle α, which is the ratio of the perpendicular and the parallel velocity components to the field line :

$$\tan\alpha = \frac{v_\perp}{v_\parallel} \tag{1}$$



1 Path around field lines

The modulus of Earths magnetic dipole moment is $M_E = 8.05 \times 10^{22}$ A m^2. Earths magnetic field can be expressed in spherical coordinates as a function of the latitude λ and the distance r to the center of the planet:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{M_E}{r^3}\left(-2\sin\lambda\vec{u}_r + \cos\lambda\vec{u}_\lambda\right) \tag{2}$$

where u_r and u_λ are unitary vectors pointing radial and polar directions respectively, with azimuthal symmetry. However, for our purposes it will be useful to express the field modulus value along one of the field lines. These lines follow the equation $r = r_{Eq}\cos^2\lambda$ where r_{Eq} is the distance from the line to the center of the Earth at Equator. Moreover, the distance r_{Eq} can also be expressed as a function of the parameter $L = \frac{r_{Eq}}{R_E}$. With this notation, we can identify a field line with the parameter L.

(a) Determine the modulus of the magnetic field B along a field line as a function of the variables λ and L. The magnetic field in the surface of the Earth at the Equator is B_E. (1.5 points)

(b) Calculate the gyrofrequency ω_g and the gyroradius r_g of the path of the protons around a field line as a function of α, λ, L, and its kinetic energy W. (2.0 points)

2 Mirror points

When a proton rotates around a field line it generates a magnetic moment which remains constant along its path. When the proton reaches some latitude λ_m the parallel velocity becomes zero and the particle starts its path backwards. This latitude is known as the mirror point.

(a) Determine the magnetic moment μ created by a proton when it rotates around a magnetic field line as a function of L, W and the pitch angle of a particle at Equator α_{Eq}. (0.8 point)

(b) Determine the relation between the mirror point latitude λ_m and α_{Eq}. Compute its value numerically when $\alpha_{Eq} = 30°$. (2.0 points)

3 Particle collision

If the mirror point lies not far from the surface of the Earth the proton collides with the particles of the atmosphere. This distance is small compared with Earth radius ($R_E = 6400$ km), so we will assume that the particles will collide when the mirror point is in the surface of the Earth.

(a) Determine the minimum equatorial pitch angle α_l, below which a proton would collide with the surface of the Earth as a function of L. (1.2 points)

(b) Determine the time that it takes for a proton to complete a whole cycle of the bouncing movement between the mirror points, with $\alpha_{Eq} = 30°$, $W = 10.0$ MeV and $L = 5.0$.

(1.2 points)

(c) Compute the length of the helical path of the proton in a whole cycle, taking into account the two movements we have considered in this problem. (0.8 point)