Problem 9: high-efficiency light emitting diode (idea contributed by Mihkel Heidelberg)

Introduction. As compared to ordinary light bulbs, light emitting diodes (LED) provide very high lighting efficiency. The reason is that the spectral energy distribution of ordinary lamps is close to black body radiation, in which case one can say that the photons are in thermal equilibrium with the black body. Then, the total energy radiated by a black body per unit area, unit time, and unit frequency interval is given by Planck's law

$$I = \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1},$$

where ν is the frequency, $h = 6.626\times10^{-34}\,\mathrm{J\cdot s}$ — the Planck constant, $c = 2.997\times10^8\,\mathrm{m/s}$ — the speed of light, $k = 1.38\times10^{-23}\,\mathrm{J\cdot K^{-1}}$ — the Boltzmann constant, and T — the temperature; note that

$$\int_0^\infty I d\nu = \sigma T^4,$$

where $\sigma = 5.678\times10^{-8}\,\mathrm{W\cdot m^{-2}\cdot K^{-4}}$ is the Stefan-Boltzmann constant. With a black body radiation, a lot of energy is wasted by radiating non-visible light. Meanwhile, LED-s can be constructed so that they radiate almost only visible light.

In recent experiments [1], it has been reported that such LED-s have been constructed which have efficiency higher that 100%. Here the efficiency is defined as the ratio of the radiated light energy to the consumed electrical energy.

Problem. Based on reasonable approximations, find what is the theoretically highest possible efficiency of a LED assuming that:
(a) the LED has a heat sink which is kept at the room temperature $T_0 = 293\,\mathrm{K}$ (via a fast enough heat exchange with the surrounding medium);
(b) the LED emits light at wavelengths smaller than $\lambda_0 = 700\,\mathrm{nm}$
(b) the surface area of the light-emitting part of the LED is $S = 1\,\mathrm{mm}^2$;
(c) the light emission power of the LED is $P = 1\,\mu\mathrm{W}$.

Hints after the first week. Since the efficiency is larger than 100%, there needs to be another source of energy (apart from the electrical energy), and with the given experimental setup, there is only one possibility: the energy is taken from the surroundings in the form of heat. This is perfectly possible, because heat energy can be easily transferred to the diode via the heat sink. However, there is a limit to how much heat can be transferred. That theoretical limit depends on the working regime of the LED. In particular, a higher light radiation power by a fixed surface area of the LED will result in a lower theoretical limit for the efficiency.

Hints after the second week. Since some heat is taken from the environment, the entropy of the environment is decreased respectively. However, entropy cannot decrease in a closed system, so at least some of the LED light needs to be radiated in the form of heat. Your first sub-task is to figure out, what is the optimal temperature of that heat.

Hints after the third week. So, the LED converts using electrical energy a certain amount of environment's heat (at T_0) into heat in the form of electromagnetic radiation at temperature T_1. This process needs to conserve energy and cannot decrease the overall entropy. These conditions put limit to the heat conversion efficiency. Note also that the density of thermal electromagnetic radiation falls rapidly with temperature, and too low values of T_1 cannot yield the required output power of the LED.

Hints after the fourth week. If we sum up what has been said above, we can conclude that the LED needs to work as a heat pump, which has maximal efficiency when operating along a reverse Carnot' Cycle. The light is emitted in thermal equilibrium with the surroundings at temperature T_1, i.e. according to the Planck's law. For maximal efficiency, it is better if no light is radiated at wavelengths longer than λ_0; this can be achieved if it has such a coating which absorbs everything (and radiates according to the Planck's law) at $\lambda < \lambda_0$, and reflects everything at longer wavelengths. The lowest acceptable temperature T_1 can be found from the requirements (b) and (c), see the problem text.

Status after the 44th IPhO. The solutions will be published on 22nd July; you can still submit a solution if you do it by 21st July, 23.59 GMT.

[1] P. Santhanam et al, *Thermoelectrically Pumped Light-Emitting Diodes Operating above Unity Efficiency*, Phys. Rev. Lett. **108**, 097403 (2012)

Correct results have been submitted by:

1. **Oliver Edtmair** (Austria), 3.330 pts;
2. **Jordan Jordanov** (Bulgaria), 3.027 pts;
3. Ly Nguyen (Vietnam), 1.111 pts;
4. Katerina Naydenova (Bulgaria), 2.144 pts;
5. **Ismael Salvador Mendoza Serrano** (Mexico), 2.274 pts;
6. **Jaemo Lim** (Korea), 2.068 pts;
7. Cyuan-Han Chang (Taiwan), 1.464 pts;
8. Dan-Cristian Andronic (Romania), 1.331 pts;
9. Joao Victor De Oliveira Maldonado (Brazil), 1.21;
10. David Stein (Germany), 1.1 pts;
11. Andres Põldaru (Estonia), 0.8 pts;
12. Szabó Attila (Hungary), 0.8 pts;
13. Kurenkov Mikhail (Russia), 1 pt;
14. Jan Ondras (Slovakia), 1 pt;
15. Kuo Pei-Cheng (Taiwan), 0.8 pts;
16. Madhivanan Elango (United Kingdom) 0.8pt.

(Bold font marks the recipients of the best solution award, $e^{1/4}$ each.)

Regarding the best-solution-award, I would have been willing to give an award for a nice and convincing explanation (based on the second law of thermodynamics), why the reversed Carnot' cycle has the highest efficiency. Also, a discussion of why the radiation needs to follow the Planck's law (which is cut off for $\lambda < \lambda_0$) would have been appreciated. While no perfect explanations were submitted, Jaemo Lim had written useful inequalities, and Ismael Salvador Mendoza Serrano has made an attempt of explaining the functioning details of such a LED, with which they both deserve a quarter of the bonus. The remaining part of the bonus is shared between Oliver Edtmair and Jordan Jordanov, who submitted their fairly solid solutions in an relatively early stage when only a minor hint was available.

The order of the attatched solutions:

1. Katerina Naydenova: an example of a very minimalistic solution;
2. Oliver Edtmair;
3. Jordan Jordanov: note how the integration is simplified by approximating $\nu \approx \nu_0^3$ (equivalent to keeping only the largest term which appears when integrating by parts, c.f. the first solution);
4. Ismael Salvador Mendoza Serrano (see comments above);
5. Jaemo Lim (see comments above).

Please let us know, which were your favourite Selection Round problems: send an e-mail to `wopho.selection@stkipsurya.ac.id` and provide an ordered list of your favourite problems (as long as you wish).

WoPhO Problem 9

$\eta = \frac{Q'}{A} = \frac{Q'}{Q'-Q}$; for $\eta_{max} \to \frac{Q}{Q'} = \frac{T}{T'} \Rightarrow \eta_{max} = \frac{1}{1 - T/T'}$

$T = 293K$, Q, A, Q', $T' = T_{eff}$ effective temp. of the radiating diode

$P = S\int_{\nu_0}^{\infty} f(\nu)\,d\nu$

for $kT \ll h\nu \to e^{h\nu/kT} \gg 1 \Rightarrow f(\nu) \approx \frac{2\pi h\nu^3}{c^2}\cdot e^{-\frac{h\nu}{kT_{eff}}}$

using that $\nu_0 = \frac{c_0}{\lambda_0} \gg 1 \Rightarrow \int_{\nu_0}^{\infty} f(\nu)\,d\nu = -\frac{2\pi k T_{eff}}{c^2}\left(\int_{\nu_0}^{\infty} 3\nu^2 e^{-\frac{h\nu}{kT_{eff}}}d\nu - \nu^3 e^{-h\nu/kT_{eff}}\Big|_{\nu_0}\right) \approx \frac{2\pi k T_{eff}\nu_0^3}{c^2}\cdot e^{-\frac{h\nu}{kT_{eff}}}$

lower by 1 order in ν $\Rightarrow$ can be neglected

$\Rightarrow e^{\frac{h\nu}{kT_{eff}}} = \frac{2\pi ck S}{P\lambda_0^3} T_{eff}$ - can be solved numerically only

$e^{\frac{20578K}{T_{eff}}} = 75838 \cdot T_{eff} \Rightarrow T_{eff} \approx 1127\,K \Rightarrow \eta_{max} = \frac{1}{1-\frac{293K}{1127K}} \approx 135\%$

Problem 9

P_e is the electrical power, P the light emission power of the LED and P_h the power taken from the surroundings in form of heat. Conservation of energy gives

$$P = P_e + P_h$$

The efficiency η of the LED is

$$\eta = \frac{P}{P_e}$$

The power radiated by the LED within a small frequency interval df is

$$dP = \varphi df$$

where φ is a function of f. In case of a black body, the radiation is in thermal equilibrium with the body. This means that the radiation is at the same temperature as the body. The energy distribution of a black body is given by

$$dP = \frac{2\pi hS}{c^2}\frac{f^3}{e^{\frac{hf}{kT}} - 1}df$$

The equation for the temperature of the radiation within a frequency interval of an arbitrary energy distribution φ is

$$\varphi = \frac{2\pi hS}{c^2}\frac{f^3}{e^{\frac{hf}{kT}} - 1}$$

The efficiency of the LED is limited by the Second Law of Thermodynamics. The LED works at maximum efficiency if the total entropy of the LED stays constant. Thus

$$\frac{P_h}{T_0} = \int \frac{dP}{T}$$

where T depends on the frequency. Combining the equations above results in

$$\eta = \frac{P}{P - P_h} = \frac{P}{P - T_0 \int \frac{dP}{T}}$$

$\varphi(f)$ is unknown.

$\varphi(\lambda) = 0$ for $\lambda > \lambda_0$ ($\varphi(f) = 0$ for $f < f_0$) and

$$\int_0^\infty \varphi df = P$$

or

$$\int_{f_0}^{\infty} \varphi df = P$$

The efficiency of a process is maximal if the process is reversible. Black body radiation is in thermal equilibrium. Therefore, $\varphi(f)$ looks like the energy distribution of a black body for $f > f_0$ and the temperature T is constant for all frequencies $f > f_0$.

$$\eta = \frac{P}{P - \frac{T_0}{T}\int dP} = \frac{T}{T - T_0}$$

T can be found from

$$\int_{f_0}^{\infty} \frac{2\pi hS}{c^2} \frac{f^3}{e^{\frac{hf}{kT}} - 1} df = P$$

$e^{\frac{hf}{kT}} \gg 1$ thus

$$\int_{f_0}^{\infty} \frac{2\pi hS}{c^2} \frac{f^3}{e^{\frac{hf}{kT}}} df \approx P$$

Partial integration gives

$$\frac{2\pi kSTe^{-\frac{hf_0}{kT}}\left(h^3{f_0}^3 + 3h^2kT{f_0}^2 + 6hk^2T^2f_0 + 6k^3T^3\right)}{c^2h^3} \approx P$$

$$\left[\frac{kT}{hf_0} + 3\left(\frac{kT}{hf_0}\right)^2 + 6\left(\frac{kT}{hf_0}\right)^3 + 6\left(\frac{kT}{hf_0}\right)^4\right] e^{-\frac{hf_0}{kT}} \approx \frac{c^2}{2\pi h{f_0}^4}\frac{P}{S} \approx \frac{{\lambda_0}^4}{2\pi hc^2}\frac{P}{S} \approx 6.42 \cdot 10^{-10}$$

$\frac{kT}{hf_0}$ can be calculated with Wolfram Alpha (http://www.wolframalpha.com).

$$\frac{kT}{hf_0} \approx 0.0543$$

$$T \approx 1120\ K$$

Setting the value of T into the expression for η gives

$$\eta = \frac{T}{T - T_0} \approx 1.35$$

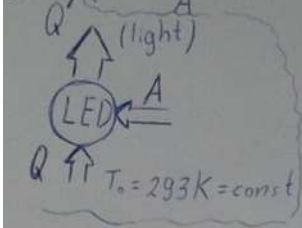
The highest possible efficiency is 135 %.

Lets mark the efficiency by "η"

$\Rightarrow \eta = \frac{A+Q}{A}$ "A" is the work done by the electrical current

"Q" is the heat pumped from the surrounding medium

"Q'" is the energy of the emited light ($Q' = Q + A$)



$$\Rightarrow \eta = \frac{Q'}{Q'-Q} = \frac{1}{1-\frac{Q}{Q'}}$$

The entropy of the LED system ~~[illegible]~~ can not decrease. So in the most efficient case $\boxed{\frac{Q}{T_0} = \frac{Q'}{T'}}$

T' is the temperature of the light-emitting part of the LED

$$\Rightarrow \eta = \frac{1}{1-\frac{T_0}{T'}}$$

so our only task is to determine the temperature "T'".

T' can be determined from the equation for the power of the LED:

$$P = S\int_{\frac{c}{\lambda_0}}^{\infty} I(\nu, T')\,d\nu = \frac{2\pi h S}{c^2}\int_{\frac{c}{\lambda_0}}^{\infty} \frac{\nu^3}{e^{h\nu/kT'}-1}\,d\nu$$

$$\nu = \frac{c}{\lambda} \Rightarrow \nu_0 = \frac{c}{\lambda_0}$$

If we assume that T' is not much more than 1000 K, we can make the approximation that: $h\nu_0 \gg kT'$

$$\Rightarrow I(\nu, T) \approx \frac{2\pi h}{c^2}\nu^3 e^{-\frac{h\nu}{kT}}$$

$$\Rightarrow P = \frac{2\pi h S}{c^2}\int_{\nu_0}^{\infty} \nu^3 e^{-\frac{h\nu}{kT'}}\,d\nu = \frac{2\pi h S}{c^2}\left(\nu_0^3\left(+\frac{kT'}{h}\right)e^{-\frac{h\nu_0}{kT'}} - \int_{\nu_0}^{\infty} e^{-\frac{h\nu}{kT'}}\left(-\frac{kT'}{h}\right)3\nu^2\,d\nu\right)$$

$$\Rightarrow \text{when } h\nu_0 \gg kT', \int_{\nu_0}^{\infty}\nu^3 e^{-\frac{h\nu}{kT'}}\,d\nu \approx \nu_0^3\left(-\frac{kT'}{h}\right)e^{-\frac{h\nu_0}{kT'}}$$

$$\Rightarrow P = \frac{2\pi h S \nu_0^3}{c^2}\left(+\frac{kT'}{h}\right)e^{-\frac{h\nu_0}{kT'}} = \frac{2\pi S \nu_0^3 kT'}{c^2}\cdot e^{-\frac{h\nu_0}{kT'}}$$

This equation can be solved only numerically

$$\Rightarrow T' \cdot e^{-\frac{20578}{T'}} = \frac{Pc^2}{2\pi S \nu_0^3 k} \approx 1{,}32 \cdot 10^{-5}\ K$$

The only solution of this equation is $T' \simeq 1130\ K$

$$\Rightarrow \boxed{\eta_{max} = \frac{1}{1-\frac{T_0}{T'}} \approx 1{,}35 = 135\%}$$

World Physics Olympiad 2013
Physics Cup

Problem 9: high-effciency light emitting diode

June 24, 2013

Ismael Salvador Mendoza Serrano

The first step is step is determining the LED's temperature with the condition that the light emission power is $P = 1\,\mu W$ (which is assumed to be the radiated power correspoding to the range of visible light, this range [1] is from $400\,nm$-$700\,nm$ corresponding from $430\,THz$ to $750\,THz$). This means that the following equation must be true:

$$P = S\int_{\nu_1}^{\nu_2} I(\nu, T)\,d\nu$$

Where ν_2 and ν_1 are frequencies corresponding to the visible spectrum. Also we have:

$$I(\nu, T) = \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT_1} - 1}$$

$$\therefore \quad P = \frac{2\pi hS}{c^2}\int_{\nu_1}^{\nu_2}\frac{\nu^3}{e^{h\nu/kT_1} - 1}d\nu$$

The integral can be simplified by making the substitutions $a = \frac{2\pi hS}{c^2}$, $b = \frac{h}{k}$, $u = \nu b/T_1 \Longrightarrow \nu = uT_1/b \Longrightarrow$ $d\nu = \frac{T_1}{b}du$, $u_1 = \nu_1 b/T_1$ $u_2 = \nu_2 b/T_1$, it results...

$$P = \frac{a}{b^4}T^4\int_{u_1}^{u_2}\frac{u^3}{e^u - 1}du$$

The temperature must be such that: (imputting corresponding values):

$$1\mu W = \left(8.72104 \times 10^{-9}\,W/K^4\right)T_1^4\int_{20646.2\,K/T_1}^{36010.9\,K/T_1}\frac{u^3}{e^u - 1}du \tag{1}$$

The integral is difficult to solve in such a way that one can leave it in terms of T_1, even then the equation is to complex to be solve for T_1. One could also approximate using Wien approximation, but even then, the resulting algebraic equation to be solved for T_1 is too complex.

What was done to solve it was to use Mathematica and use a trial-and-error method of imputting different temperatures and evaluating until the right hand side of the eq. (1) becomes the left hand side of it.

The result was $T_1 \approx 1120K$, this is of course the temperature of the photon field of the LED.

Solving for T_1 directly is too difficult, but by looking at the graph[1] of $\int \frac{x^3}{e^x - 1}dx$, one could think that there is only one solution, and by using $T_1 = 1120K$ in (1), one can demostrate that $1120\,K$ is a solution...

$$P = \left(1.37 \times 10^4 W\right)\int_{18.43}^{32.2}\frac{u^3}{e^u - 1}du$$

Focusing on the integral...

$$\begin{aligned} I' &= \int \frac{u^3}{e^u - 1}du \\ &= \int u^3\frac{e^{-u}}{1 - e^{-u}}du \end{aligned}$$

[1]https://dl.dropboxusercontent.com/u/38822094/Importante/Wopho/Led/graph12.JPG

But, we can identify the result of the geometric sum:

$$\sum_{1}^{\infty} e^{-ku} = \frac{a}{1-r} = \frac{e^{-u}}{1-e^{-u}}$$

Which allows us to write...

$$I^{'} = \int u^3 \left(\sum_{1}^{\infty} e^{-ku} \right) du$$

Now we make the substitutions $s = ku \implies du = \frac{ds}{k}$ to write...

$$\begin{aligned} I^{'} &= \int \frac{s^3}{k^3} \left(\sum_{1}^{\infty} e^{-s} \right) \frac{ds}{k} \\ &= \sum_{1}^{\infty} \frac{1}{k^4} \int s^3 e^{-s} ds \\ &= -\sum_{1}^{\infty} \frac{1}{k^4} \Gamma(4, s) \mid_{s_1}^{s_2} \end{aligned}$$

Where k is an integer, Γ represents the gamma function [5]; and $s_2 = (32.2)\, k$ and $s_1 = (18.43)\, k$ are the corresponding limits of the integral, then...

$$\begin{aligned} I^{'} &= -\sum_{1}^{\infty} [e^{-32.2k} \left((32.2)^3 k^{-1} + 3\,(32.2)^2 k^{-2} + 6\,(32.2)\, k^{-3} + 6k^{-4} \right) \\ &\quad - e^{-18.43k} \left((18.43)^3 k^{-1} + 3\,(18.43k)^2 k^{-2} + 6\left(18.43k^{-3}\right) + 6k^{-4} \right)] \\ &= 7.299 \times 10^{-11} \implies P = \left(1.37 \times 10^4 W\right)\left(7.299 \times 10^{-11}\right) \approx 1.00 \times 10^{-6}\, W \end{aligned}$$

When the LED surpasses unit efficiency it works as a thermoelectric cooler operating between its lattice and its photon field [2], it's assume that the maximum theorethical efficiency of the LED is determined by the efficiency of the reverse Carnot. If the lattice is directly connected to the heat sink, then reverse carnot efficiency [3] is:

$$\eta_{rc} = \frac{T_o}{T_1 - T_o} = \frac{293K}{1120K - 293K} \approx .354 \tag{2}$$

The reason the LED can achieve higher than unity efficiency is the following: The LED works in such a way that at the junction of the p-n conductors, the electrons and holes get together, and when this happens, the electrons fall from a higher energy level to a lower one in such a way that they emit a photon of energy proportional to the change of energy of electrons. The energy for the electrons and holes in a not-over-unit efficiency LED is obtained only from the electrical power, but in this case, the extra energy comes from the Peltier heat exchange[2], which manifests it self when the LED acts as a thermoelectric cooler, and the Peltier effect is driven also by the current but occurs when you have two different conductors in an electrified junction (like p-n) . So the thermodynamic representation of this:

[2]It's interesting to note, that the Peltier effect isn't appreciated before the conditions of over unit efficiency are met, since it's "blocked" by the Joule effect.

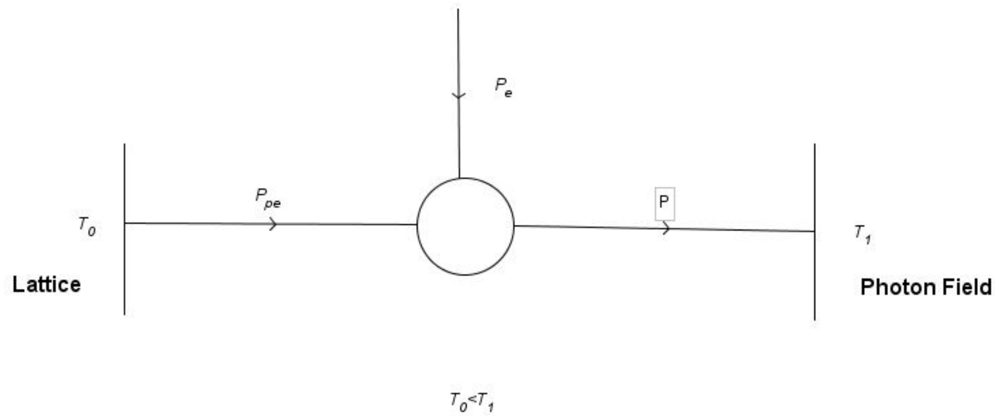


Figure 1: Thermodynamic representation of LED acting as a thermodynamic cooler.

Where P_{pe} is the power of heat transfer from Peltier effect, P_e is the electrical power, P is the resultant optical power (which makes sense, since the total energy available to the photons is the total input). The efficiency of the cycle becomes:

$$\begin{aligned}\eta_0 &= \frac{\mid P_{pe} \mid}{\mid P \mid - \mid P_{pe} \mid} \\ &= \frac{\mid P \mid - \mid P_e \mid}{\mid P_e \mid} \\ \eta_0 &= \frac{\mid P \mid}{\mid P_e \mid} - 1 \end{aligned} \tag{3}$$

But $\frac{|P|}{|P_e|}$ is the efficiency of the LED, and when the efficiency of the LED is maximum when the efficiency of the cycle is maximum (carnot cycle). So joining equations (2) and (3) gives...

$$\begin{aligned}\eta_0 &= \eta_{rc} = \eta - 1 \\ \eta &= 1 + .354 \\ \eta &\approx 1.354\end{aligned}$$

Reference:

- [1] Walker, Jearl, David Halliday, and Robert Resnick. Fundamentals of Physics. Hoboken, NJ: Wiley, 2008. Print.
- [2] P. Santhanam et al, Thermoelectrically Pumped Light-Emitting Diodes Operating above Unity Eciency, Phys. Rev. Lett. 108, 097403 (2012)
- [3] Formulas for IPHO, J. Kalda, March 16, 2012
- [4] Gray, Dodd. Thermal Pumping of Light-Emitting Diodes. August 2011
- [5] Weisstein, Eric W. "Gamma Function." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com

WoPhO Selection Round-Jaemo Lim

Problem 9. High-efficiency light emitting diode

1.

By the third and fourth assumption, light emission power per unit area of LED is $\frac{P}{S}=1\,\mathrm{W/m^2}$.

By second assumption, LED emits light at wavelengths $\lambda \leqq 700\,\mathrm{nm}$, which is $\nu \geqq \nu_0 = \frac{c}{\lambda_0} = 4.28\times 10^{14}\,\mathrm{s}^{-1}$. So $\frac{P}{S} \geqq \int_{\nu_0}^{\infty} \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1}d\nu$.

Since $\frac{h\nu_0}{kT} > 6.8$ when $T \leqq 3000K$(the temperature of LED would not be higher than 1000K because the components will not stand this temperature), $e^{\frac{h\nu_0}{kT}} \sim 900 \gg 1$ and we can approximate $e^{h\nu/kT}-1 \approx e^{h\nu/kT}$.

Therefore, $\frac{P}{S} = \int_{\nu_0}^{\infty} \frac{2\pi h}{c^2}\nu^3 e^{-\frac{h\nu}{kT}} d\nu = 1\,\mathrm{W/m^2}$.

Definite integral $\int_{\nu_0}^{\infty} \frac{2\pi h}{c^2}\nu^3 e^{-\frac{h\nu}{kT}} d\nu$ can be solved with integration by parts.

$$\frac{P}{S} = \int_{\nu_0}^{\infty} \frac{2\pi h}{c^2}\nu^3 e^{-\frac{h\nu}{kT}} d\nu = \frac{2\pi h}{c^2}\left[-\frac{kT}{h}\nu^3 e^{-\frac{h\nu}{kT}}\right]_{\nu_0}^{\infty} + \frac{2\pi h}{c^2}\frac{3kT}{h}\int_{\nu_0}^{\infty}\nu^2 e^{-\frac{h\nu}{kT}} d\nu$$

$$= \cdots = \frac{2\pi h}{c^2} e^{-\frac{h\nu_0}{kT}}\left(\frac{kT}{h}\nu_0^3 + 3\left(\frac{kT}{h}\right)^2\nu_0^2 + 6\left(\frac{kT}{h}\right)^3\nu_0 + 6\left(\frac{kT}{h}\right)^4\right)$$

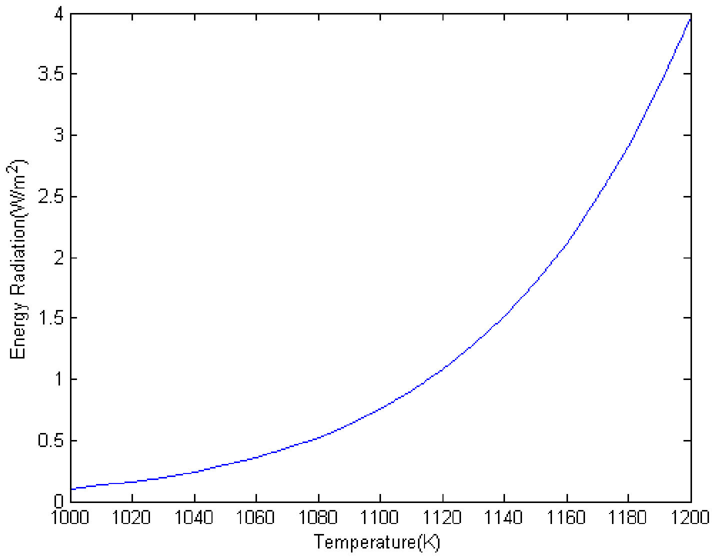


Figure 1 Energy radiation per unit area according to temperature

T(K)	P/S(W/m^2)
1115	0.9871
1116	1.0046
1117	1.0223

Table 1 Energy radiation per unit area near 1W/m^2.

2.

By plot of this equation in figure 1 and calculation in table 1, it can be found out that to satisfy $\frac{P}{S} \geqq \int_{\nu_0}^{\infty} \frac{2\pi h}{c^2} \frac{\nu^3}{e^{h\nu/kT_1} - 1} d\nu$, the temperature should satisfy $T_1 \geqq 1116\mathrm{K}$.

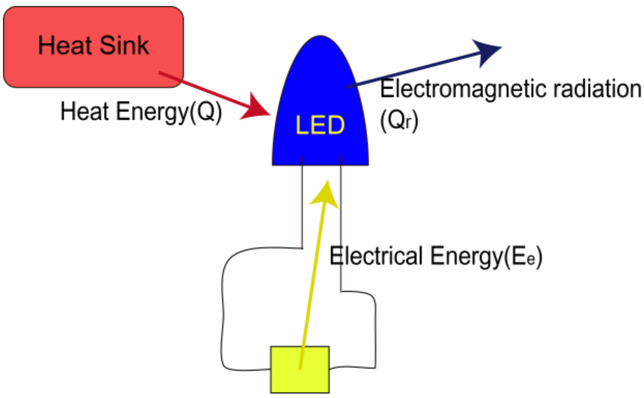


Figure 2 Energy transfer model of LED

The transferred energy in figure 2 is heat energy, electrical energy, and electromagnetic radiation. Also, there can be some energy loss in LED, but it will decrease the efficiency of LED so I will ignore this energy loss to get the maximum efficiency.
By energy conservation, $Q_r = Q + E_e$.
The entropy must increase in a closed system. By heat transfer from heat sink, $\Delta S_1 = -\frac{Q}{T_0}$ occurs. Radiation increases total entropy of the system, $\Delta S_2 = \frac{Q_r}{T_1}$.

Therefore, $\Delta S = \Delta S_1 + \Delta S_2 = -\frac{Q}{T_0} + \frac{Q_r}{T_1} \geqq 0$, $\frac{T_0}{T_1} \geqq \frac{Q}{Q_r}$

Since the definition of efficiency is $\epsilon = \frac{Q_r}{E}$, $\epsilon = \frac{Q_r}{Q_r - Q} = \frac{1}{1 - \frac{Q}{Q_r}}$.

Using $\frac{T_0}{T_1} \geqq \frac{Q}{Q_r}$, $1 - \frac{Q}{Q_r} \geqq 1 - \frac{T_0}{T_1}$ and $\epsilon = \frac{Q_r}{Q_r - Q} = \frac{1}{1 - \frac{Q}{Q_r}} \leqq \frac{1}{1 - \frac{T_0}{T_1}} = \frac{T_1}{T_1 - T_0}$.

Plot of maximim efficiency is in figure 3.

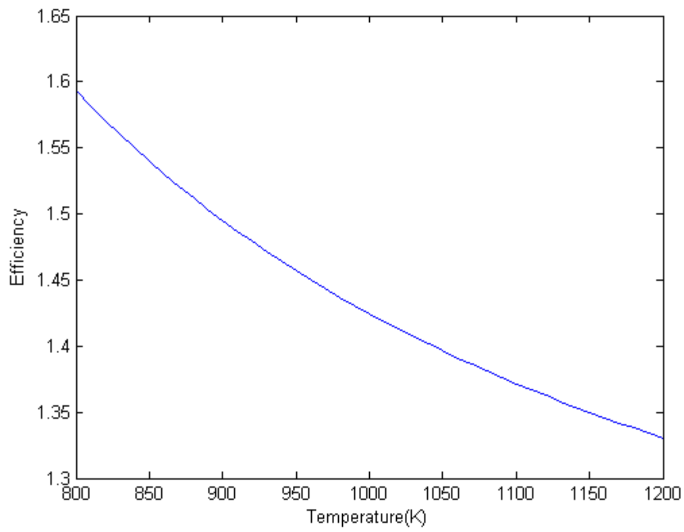


Figure 3 Maximim efficiency of LED according to temperature of radiation

The efficiency of LED decreases as temperature increases, so the LED has the highest efficiency when the temperature is the lowest. Using the condition $T_1 \geqq 1116\mathrm{K}$, maximum efficiency of LED

$$\epsilon_{\max} = \frac{1116}{1116 - 298} = 1.36 = 136\%.$$

3.

If the light radiation power per unit area gets larger, the temperature of LED should increase, so efficiency of LED would decrease as in figure 3. Maximum efficiency is calculated in an ideal condition(it does not consider energy loss in electrical circuit, there is no change in entropy, radiation energy is same with black body radiation). Therefore it would be possible to experimentally make LED that has higher efficiency than 100%.

$$\therefore \epsilon_{\max} = 136\%$$